VFAST Transactions on Mathematics
http://vfast.org/journals/index.php/VTM@ 2019, ISSN(e): 2309-0022, ISSN(p): 2411-6343
Volume 7, Number 1, January-December, 2019 pp: 22-34

More Modular Arithmetic in Five Regular Partitions by Jacobi Triple Product Formula

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In a paper, Calkin et al., Divisibility properties of the 5-regular and 13-regular partition functions, Integers 8 (2008), [#A60], authors have discussed interesting properties for 5-regular partition functions of integers. In the continuation of this paper, we have obtained and conjectured various interesting results. In this note, we use nothing more than Jacobi's triple product identity to obtain results for 5-Regular Partitions that are stronger than those obtained by Calkin and his collaborators. The motivation for this paper is an observation that some generating functions of 5-Regular partitions are congruent to functions related to the Ramanujan's Q-series.

1. INTRODUCTION

Number Theory is a branch of pure mathematics, devoted primarily to the study of integers. It is sometimes called The queen of Mathematics, because of its foundational space in the discipline. Q-series is one of the most important topic of analytic number theory. Historically, research in q-series has not always been appreciated. The beautiful and useful results in q-series are now cherished by many researchers. This paper integrants recent developments and related applications in q-series with a historical development of the field focusing on major breakthroughs. It is our great desire that the introduction to q-series will inspire formerly to new readers. The q-series disease, in the words of Richard Askey who has suffered from this disease for several decades, is less popular and hard to treat like other mathematics. Now, we give the formal definition of q-series.

Definition 1.1. A q-series is such a series which contains factors in q. This is expressed as;

$$(a;q)_n = (1-a)(1-aq)(1-aq^2)\cdots(1-aq^{n-1}), \tag{1.1}$$

for $n \geq 0$ and $(a;q)_0 = 0$. A q-series is such a series which contains factors in q. This is expressed as;

$$(a;q)_n = (1-a)(1-aq)(1-aq^2)\cdots(1-aq^{n-1}), \tag{1.2}$$

for $n \geq 0$ and $(a;q)_0 = 0$

Definition 1.2. Another representation of q-series is as under:

$$(a;q)_n = \prod_{k=1}^{n}(1-aq^{k-1})(a;q)_\infty = \prod_{k=1}^{\infty}(1-aq^{k-1}). \tag{1.3}$$

When n approaches to infinity then we use the symbol $(a;q)_\infty$, which is called a q- pochhammer symbol, introduced by Andrews in 1986. We can express the infinite q products as:

$$(a;q)_\infty = \lim_{n\to\infty}(a;q)_n, \qquad |q| < 1. \tag{1.4}$$

Q-series obey beautifully sets of properties and arise naturally in the theory of partitions of integers , as well as in many fields of mathematical Physics, especially those enumerating possible numbers of configurations or states on a Lattice. Q-series has numerous applications in Combinatorics, Number theory, Analysis, Physics and Computer

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