2.1 Example

To draw a probabilistic graph associated with a joint probability distribution, consider a joint probability distribution.

.	y_1	y_2	y_3
x_1	$\frac{3}{28}$	$\frac{6}{28}$	$\frac{1}{28}$
x_2	$\frac{9}{28}$	$\frac{6}{28}$	0
x_3	$\frac{3}{28}$	0	0

The probabilistic graph corresponding to above joint probability distribution is given below.

Figure 1: Parabolistic Graph

3 Probabilistic Picture Graph

Let $G^* = (V^*, E^*)$ be a probabilistic graph. A pair $G = (V, E)$ is called probabilistic picture graph on G^* with vertex set $V = (X, Y) = \{v_1 = (x_1, y_1), v_2 = (x_1, y_2), ..., v_n = (x_n, y_n)\}$ and edge set E such that for each pair of vertices $v_i = (x_i, y_i)$ and $v_j = (x_j, y_j)$, we have, $e(v_i v_j) \leq \min\{e(x_i y_i) e(x_j y_j)\}$ i,j=1,2,3,...,n.

3.1 Example

Probabilistic picture graph corresponding to the probabilistic graph in shown below.

Figure 2: Parabolistic Picture Graph

4 Strong Probabilistic Graph

A probabilistic picture graph $G = (V, E)$ is defined as strong probabilistic picture graph, if $e(v_i, v_j) = \min\{e(x_i y_i), e(x_j y_j)\}, \quad for \quad all \quad e \in E$ Where $v_i = (x_i, y_i)$, $v_j = (x_j, y_j) \in V, i, j = 1, 2, 3, ..., n.$

4.1 Example

An example of strong probabilistic graph is shown below

Figure 3: Strong parabolistic picture graph

5 Complete Probabilistic Picture Graph

A probabilistic picture graph $G = (V, E)$ is defined as complete probabilistic picture graph, if $e(v_i, v_j) = \min\{e(x_i y_i), e(x_j y_j)\}, i, j = \{1, 2, 3, ..., n\}$ for all $v_i = (x_i, y_i)$, $v_j = (x_j, y_j) \in V$.

6 Remark

Every complete probabilistic picture graph is strong probabilistic picture graph but not conversely.

7 A Path Probabilistic Picture Graph

A path P in a probabilistic picture graph $G = (V, E)$ is a sequence of different adjacent vertices $v_0, v_1, ..., v_k$.
Here, k represents the length of path.

7.1 Definition

Let $G = (V, E)$ be a probabilistic picture graph.If two nodes v_0 and v_k are connected by a path of length k. Then $P^k(v_0, v_k)$ for the path is described as $P^k(v_0, v_k) = e(v_0, v_1) \wedge e(v_1, v_2) \wedge ... \wedge e(v_{k-1}, v_k)$ Where $P^k(v_0, v_k)$ represents the value of the path of length k between any two vertices v_0 and v_k. Moreover, different paths between v_0 and v_k of different lengths have different values. Let $P^\infty(v_i, v_j)$ be the strength of connectedness between the two nodes v_i and v_j of a probabilistic picture graph. Then $P^\infty(v_i, v_j)$ is defined as follows $P^\infty(v_i, v_j) = sup\{P^k(v_i, v_j) | k = 1, 2, ...\}, i, j = 1, 2, ..., n$ Where superimum is used to find maximum value.

7.2 Example

Consider a connected probabilistic picture graph as shown in figure-4 and the $v_0 - v_2$ paths are

$$P_1 : v_0 - v_2 \quad with \quad value \quad P^1(v_0, v_2) = 0.1$$

$$P_2 : v_0 - v_1 - v_2 \quad with \quad value \quad P^2(v_0, v_2) = 0.1$$

$$P_3 : v_0 - v_3 - v_2 \quad with \quad value \quad P^3(v_0, v_2) = 0.1$$

$$P_4 : v_0 - v_1 - v_3 - v_2 \quad with \quad vlue \quad P^4(v_0, v_2) = 0.2$$

$$P_4 : v_0 - v_3 - v_1 - v_2 \quad with \quad value \quad P^4(v_0, v_2) = 0.1$$

Then by routine computation of connectedness between the nodes v_0 and v_2 of probabilistic picture graph is

$$P^{\infty}(v_0, v_2) = sup\{0.1, 0.1, 0.1, 0.2, 0.1\} = 0.2$$

.

8 Connected Probabilistic Picture Graph

Let $G = (V, E)$ be a probabilistic picture graph. Then G is said to be connected if, for every pair of vertices v_i ,v_j, we have $\mathbf{P}^{\infty}(v_i, v_j) > 0, i, j = \{1, 2, ..., n\}$.

8.1 Example

since values for all paths between all vertices are greater than zero. Hence strength of connectedness is also greater than zero.Therefore graph in figure-4 is connected.

9 Complement of Probabilistic Picture Graph

The complement of a probabilistic picture graph $G = (V, E)$ is a probabilistic picture graph $G^c = (V^c, E^c)$ if and only if for each pair of vertices, we have $V^c = V$ and $E^c(v_i, v_j) = 1 - E(v_i v_j), i, j = 1, 2, ..., n$.

9.1 Example

Figure 6: Parabolistic Picture Graph

Complement of above figure-6 is given below,

Figure 7: Compliment of Parabolistic Picture Graph

Also,

$$(G^c)^c = G$$

.

10 Degree of a vertex

Let $G = (V, E)$ be a probabilistic picture graph on G^*. The degree of a vertex u is defined as $deg(u) = \sum_{v_i \in V, u \neq v_i} e(u, v_i)$.

11 Order

The number of vertices in probabilistic picture graph is defined as the order.

12 Size of a probabilistic picture graph

The size of probabilistic picture graph is defined as $S(G) = \sum_{u \neq v} e(u, v), \forall u, v \in V$.

13 Operations on Probabilistic Picture Graph

In this section, we define operations on probabilistic picture graph. These are given below,

13.1 Cartesian product

Let $G_1 = (V_1, E_1)$ and $G_2 = (v_2, E_2)$ be two probabilistic picture graphs on $G_1^* = (V_1^*, E_1^*)$ and $G_2^* = (V_2^*, E_2^*)$ respectively. The cartesian product $G_1 \times G_2$ of probabilistic picture graphs G_1 and G_2 is defined by $G = (V, E)$,where $V = V_1 \times V_2$ and $E = \{(x, x_2), (x, y_2) | x \in V_1, x_2y_2 \in E_2\} \cup \{(x_1, z)(y_1, z) | z \in V_2, x_1y_1 \in E_1\}$ respectively,
which satisfies the following:

$1)\forall x \in V_1 \quad and \quad \forall (x_2y_2) \in E_2$

$e((x, x_2)(x, y_2)) = e_2(x_2y_2)$

$2)\forall z \in V_2 \quad and \quad \forall (x_1y_1) \in E_1$

$e((x_1, z)(y_1, z)) = e_1(x_1y_1)$.

13.2 Composition

The composition G_1G_2 of two probabilistic picture graphs $G_1 = (V_1, E_1)$ and $G_2 = (V_2, E_2)$ defined as $G = (V, E)$ with vertex set $V = V_1 \times V_2$ and edge set $E = \{(x, x_2)(x, y_2) | x \in V_1, x_2y_2 \in E_2\} \cup \{(x_1, z)(y_1, z) | z \in V_2, x_1y_1 \in E_1\} \cup \{(x_1, x_2)(y_1, y_2) | x_2 \neq y_2, x_1y_1 \in E_1\}$ respectively,
which satisfies the following:

$1)\forall x \in V_1 \quad and \quad (x_2y_2) \in E_2$

$e((x,x_2)(x,y_2)) = e_2(x_2y_2)$

$2)\forall z \in V_2 \quad and \quad \forall(x_1y_1) \in E_1$

$e((x_1,z)(y_1,z)) = e_1(x_1y_1)$

$3)\forall x_2,y_2 \in V_2, x_2 \neq y_2 \quad and \quad \forall(x_1y_1) \in E_1$

$e((x_1,x_2)(y_1,y_2)) = e_1(x_1y_1) \quad or \quad e_2(x_2,y_2) \quad if \quad ,x_1 \neq y_1.$

13.3 Union

The union $G_1 \cup G_2$ of two probabilistic picture graphs $G_1 = (V_1, E_1)$ and $G_2 = (V_2, E_2)$ is defined as $G = (V, E)$, where vertex set $V = V_1 \cup V_2$ and edge set $E = E_1 \cup E_2$ respectively,
which satisfies the following:

$1)e(xy) = e_1(xy) \quad if \quad xy \in E_1 \quad and \quad xy \notin E_2$

$2)e(xy) = e_2(xy) \quad if \quad xy \in E_2 \quad and \quad xy \notin E_2$

$3)e(xy) = e_1(xy) \wedge e_2(xy) \quad if \quad xy \in E_1 \cap E_2.$

13.4 Direct product

The direct product $G_1 * G_2$ of two probabilistic picture graphs $G_1 = (V_1, E_1)$ and $G_2 = (V_2, E_2)$ is defined as pair $G = (V, E)$, where vertex set is $V = V_1 * V_2$ and edge set is $E = \{(x_1,x_2)(y_1,y_2)|x_1y_1 \in E_1, x_1y_2 \in E_2\}$.
Which satisfies the following:

$1)e((x_1,x_2)(y_1,y_2)) = e_1(x_1y_1) \vee e_2(x_2y_2).$

13.5 Lexicographical product

The lexicographical product $G_1.G_2$ of two probabilistic picture graphs $G_1 = (V_1, E_1)$ and $G_2 = (v_2, E_2)$ is defined as $G = (V, E)$, where vertex set is $V = V_1 \times V_2$ and edge set is $E = \{(x,x_2)(x,y_2)|x \in V_1, x_2y_2 \in E_2\} \cup \{(x_1,x_2)(y_1,y_2)|x_1y_1 \in E_1, x_2y_2 \in E_2\}$.
Which satisfies the following:

$1)\forall x \in V_1 \quad and \quad \forall(x_2y_2) \in E_2$

$e((x,x_2)(x,y_2)) = e_2(x_2y_2)$

$2)\forall(x_1y_1) \in E_1 \quad and \quad \forall(x_2y_2) \in E_2$

$e((x_1,x_2)(y_1,y_2)) = e_1(x_1y_1) \vee e_2(x_2y_2).$

13.6 Strong product

The strong product G_1G_2 of two probabilistic picture graphs $G_1 = (V_1, E_1)$ and $G_2 = (V_2, E_2)$ is defined as the pair $G = (V, E)$,where the vertex set is $V = V_1 \times V_2$ and edge set is $E = \{(x, x_2)(x, y_2)|x \in V_1, x_2y_2 \in E_2\} \cup \{(x_1, z)(y_1, z)|z \in V_2, x_1y_1 \in E_1\} \cup \{(x_1, x_2)(y_1.y_2)|x_1y_1 \in E_1, x_2y_2 \in E_2\}$.
Which satisfies the following:

$1)\forall x \in V_1 \quad and \quad \forall(x_2y_2) \in E_2$

$e((x,x_2)(x,y_2)) = e_2(x_2y_2)$

$2)\forall(x_1y_1) \in E_1 \quad and \quad \forall(x_2y_2) \in E_2$

$e((x_1,x_2)(y_1,y_2)) = e_1(x_1y_1) \vee e_2(x_2y_2)$

$3)\forall(x_1y_1) \in E_1 \quad and \quad \forall z \in V_2$

$e((x_1,z)(y_1,z)) = e_1(x_1y_1).$

14 APPLICATION

The following well-known applications can be used for probabilistic picture graphs as well in the same spirit as for ordinary picture graphs. Traditionally

The probability for the joint probabilities lies between 0 and 1. Using a real time data for population over an entire country is given by,

	$Malaysia(y_1)$	$Sirilanka(y_2)$	$Thailand(y_3)$	$Nepal(y_4)$	$Zambia(y_5)$
$India(x_1)$	$\frac{3}{10}$	$\frac{1}{2}$	$\frac{4}{5}$	$\frac{9}{10}$	$\frac{9}{10}$
$US(x_2)$	$\frac{7}{10}$	$\frac{3}{5}$	$\frac{2}{5}$	$\frac{9}{10}$	$\frac{4}{5}$

The probabilistic picture graph is more efficient to model the probability of success in the joint venture of different countries. That how much the joint venture is strong between any two countries. If edge in probabilistic graph corresponding to the above table has value 1,it shows 100 percent success but its not the reality. Greater the value of arc stronger the joint venture between corresponding countries. Here vertices are the different countries between which we want to examine the joint venture's failure or succcess. Also, complement of probabilistic picture graph shows the percentage of losses of the joint venture between corresponding countries. The probabilistic picture graph shows success of joint venture between different pair of countries. In other words a single vertex shows the pair of countries.

The probabilistic picture graph for above table is

Figure 8:Parabolistic Picture Graph

References

[1] Zadeh, Lotfi A. Fuzzy sets. Information and control 8, no. 3 (1965) 338-353.