ANALYSIS OF ELLIS FLUID IN WIRE COATING

MUHAMMAD AYAZ, TAZA GUL, SAEED ISLAM, MUHAMMAD ALTAZ KHAN

Department of Mathematics, Abdul Wali Khan, University Mardan, Khyber Pakhtunkhwa, Pakistan

Revised May 2015

ABSTRACT. *The focus of the present work is concerned with the study of wire coating analysis using Ellis fluid. Using the binomial series method, closed form solution is obtained and further in the closed form of solution, we discussed a special case for the parameter n , and obtained the expressions for axial velocity, shear stress, average velocity, volume flow rate and temperature distribution. Thickness of the coated wire and force on the total wire are also calculated. The effect of different parameters of interest on the variation of the velocity profile, volume flow rate, shear stress, radius of coated wire and force on the total wire are discussed and presented graphically.*

Keywords: Wire Coating, Ellis Fluid, Pressure Type Die, Binomial Series Method.

1. Introduction. Wire coating is used for the purpose of high and low voltage, for the protection of humans, for the processing of signals such as cable and telephone wires and for the protection against corrosion. It is performed due the drag of wire in the coating unit filled with molten polymers, and hence wire is coated due to the shear stress between the wire and molten polymer. Wire coating is one of the important chemical process in which different types of polymers are used. The coating of wire depends on geometry of die, viscosity of fluid, temperature of wire and molten polymer used for coating. Wire coating have many applications in the field of chemical and industrial engineering. Many authors have studied wire coating phenomena.

The basic concept of modeling the wire coating for viscous fluid is given by Denn and Middleman [1, 2]. Akhter and Hashmi [3, 4] have studied wire coating using power law fluid and have investigated the effect of change in viscosity. Fenner and Williams [5] carried out an analysis of the flow in the tapering section of a pressure type die. Siddiqui et al [6] studied wire coating extrusion in a pressure type die in the flow of third grad fluid. They obtained the numerical solutions for the pressure and velocity profiles in the die. Sajjid et al [7] studied the wire coating with oldroyd 8-constant fluid and give the solution for velocity field in the form of series. Gagley and Story [8] provided numerical simulations for Newtonian fluid in the form of dimensionless parameters characterizing the wire speed, die dimension, radial position, shear rate and melt viscosity. The analysis of drag flow of coated polymer outside pressure type die was carried out by Kasajima and Katsuhiko Ito [9]. They derived the expressions for velocity and temperature field. Moreover, they found average velocity, volume flow rate and discussed some cases for constant temperature on the boundaries. Han and Rao [10] discussed the rheology of wire coating extrusion. Casewell and Tanner [11] made first attempt to analyze the isothermal flows of Newtonian and power law fluids inside the wire coating dies, without making use of lubrication approximation theory (LAT). Roy and Dutt [12] further investigated and suggested the theory for wire coating by withdrawal from a power law fluid and tested with experimental data on Newtonian and pseudoplastic fluids. Evan Mitsoulis [13] studied fluid flow and heat transfer in wire coating. Recently, Rehan et al [14] made the first attempt to analyze the isothermal flows of unsteady second grad fluid inside the wire coating die with oscillating boundary condition using the Optimal Homotopy Asymptotic Method. Rehan et al [15] studied the wire coating analysis with oldroyd 8-constant fluid by Optimal Homotopy Asymptotic Method, and obtained the solution for velocity field. Rehan et al [16] also studied the exact solution of differential equation arising in the wire coating analysis of an unsteady second grad fluid. Mckelvey [17] and Paton et al [18] have analyzed the flow of Newtonian and power law fluid models in the

wire coating analysis, and obtained the expressions for the flow rate, shear rate and velocity distribution along the radial direction.

According to the best of our knowledge the mathematical modeling using an Ellis fluid model of wire coating process with no-slip boundary conditions has not been studied. As the Ellis fluid has many industrial applications like waterborne coating and metallic automotive base coating where polymeric suspensions are used, lubricating greases, psueodoplastic fluids and drilling fluids are its industrial applications [20, 21]. Thus we modeled the wire coating process by considering Ellis fluid as melt and develop the resulting differential equation by using Binomial series technique. The non-linear differential equations governing the model are made dimensionless and solved for velocity and temperature distribution. We derive closed form of solution in dimensionless form for velocity field, shear stress, volume flow rate, thickness of coated wire, force on total wire and temperature distribution.

2. Basic Governing Equations. The basic laws of conservation of mass, conservation of momentum governing the flow of incompressible Ellis fluid model with thermal effect is

$$\nabla \cdot V = 0, \quad (1)$$

$$\rho \frac{Dv}{Dt} = -\nabla P + \text{div } S + \rho f, \quad (2)$$

$$\rho C_p \frac{D\theta}{Dt} = k \nabla^2 \theta + S \cdot \nabla V, \quad (3)$$

where ρ is constant density, V is velocity vector, f is body force, p is pressure, S is the extra stress tensor and $\frac{D}{Dt}$ is the material derivative. As discussed in [19] the stress tensor defining the Ellis fluid is given by

$$S = \eta A_1, \quad (4)$$

Where

$$\frac{1}{\eta} = \frac{1}{\eta_0} \left[1 + \left| \frac{\frac{1}{2}(S:S)}{\frac{S_1}{2}} \right|^{\alpha-1} \right], \quad (5)$$

And

$$A_1 = \nabla V + (\nabla V)^T, \quad (6)$$

Where η is the apparent viscosity, η_0 is zero shear viscosity, $\frac{S_1}{2}$ represents the value of shear stress at which the apparent viscosity has dropped to half its zero shear value, and lastly the index α is a measure of the degree of shear thinning behavior that is the greater the value of α greater is the extent of shear thinning.

3. Formulation and solution of the problem. Consider steady, parallel, laminar flow of an incompressible Ellis fluid flowing in stationary cylindrical die of length L and of radius R_d . The wire of radius R_w is translating along the center line of the die with uniform velocity U_w as shown in the **Fig. 1** below. The wire and die are taken to be concentric. The fluid is acted upon by a constant pressure gradient $\frac{dp}{dz}$ in the z -direction. The drag of wire and pressure gradient along the axial direction is responsible for the flow of fluid. We choose the cylindrical coordinate system (r, θ, z) such that z -axis is along the centre of the die and r -axis is normal to z -axis.

Fig. 1 : Schematic profile of wire coating in pressure type die.

The boundary conditions for the problem on the velocity field and temperature distribution are given by

$$w(R_w) = U_w, \quad w(R_d) = 0, \quad (7)$$

$$\theta(R_w) = \theta_w, \quad \theta(R_d) = \theta_d. \quad (8)$$

The fluid velocity field and temperature distributions are considered as

$$\mathbf{V} = [0, 0, w(r)], \quad S = S(r), \quad \theta = \theta(r), \quad (9)$$

Using the assumptions continuity **Eqn. 1** satisfied identically, the momentum **Eqn. 2** and energy **Eqn. 3** in the absence of body forces reduces to

$$\frac{\partial P}{\partial r} = 0, \quad (10)$$

$$\frac{\partial P}{\partial z} = 0, \quad (11)$$

$$\frac{\partial P}{\partial z} = \frac{1}{r} \frac{d}{dr} (r S_{rz}), \quad (12)$$

$$k \left(\frac{d^2}{dr^2} + \frac{1}{r} \frac{d}{dr} \right) \theta + S_{rz} \frac{dw}{dr} = 0, \quad (13)$$

From **Eqns. 10-12** it is concluded that 'p' is function of 'z' only, also assume that there is no variation in pressure gradient along the axial direction. Thus we have $\frac{\partial P}{\partial z} = B$, where

B is a constant.

Thus **Eqn. 12** reduces to the form

$$\frac{1}{r} \frac{d}{dr} (r S_{rz}) = B,$$

Integrating w.r.t 'r' we get

$$S_{rz} = \left(\frac{B}{2} r + \frac{k_1}{r} \right), \quad (14)$$

Where k_1 is an arbitrary constant of integration.

Now by the use of **Eqn. 9**, in the constitutive equations, the combination of **Eqn. 4** and **Eqn. 5** is given by

$$\frac{dw}{dr} = \frac{1}{\eta_0} \left[1 + \left| \frac{S_{rz}}{S_1} \right|^{\alpha-1} \right] \cdot S_{rz}, \quad (15)$$

Substituting **Eqn. 14** in **Eqn. 15** and for relaxing the absolute condition, we have

$$\frac{dw}{dr} = \frac{1}{\eta_0} \left(\frac{B}{2} r + \frac{k_1}{r} \right) + \frac{k_1^\alpha}{\eta_0 r^\alpha S_1^{\alpha-1}} \left(1 + \frac{B}{2k_1} r^2 \right)^\alpha, \quad (16)$$

Using the Binomial series formula and taking $\left| \frac{B r^2}{2k_1} \right| < 1$, we obtain

$$\frac{dw}{dr} = \frac{B}{2\eta_0} r + \frac{k_1}{\eta_0 r} + \frac{k_1^\alpha}{\eta_0 S_1^{\alpha-1}} r^{-\alpha} + \frac{\alpha B k_1^{\alpha-1}}{2\eta_0 S_1^{\alpha-1}} r^{2-\alpha} + \frac{\alpha(\alpha-1)}{2!} \frac{B k_1^{\alpha-2}}{4\eta_0 S_1^{\alpha-1}} r^{4-\alpha} + \frac{\alpha(\alpha-1)(\alpha-2)}{3!} \frac{B k_1^{\alpha-3}}{8\eta_0 S_1^{\alpha-1}} r^{6-\alpha} + \dots, \quad (17)$$

Now introducing the following dimensionless parameters in **Eqn. 17**

$$w^* = \frac{w}{U_w}, \quad r^* = \frac{r}{R_w}, \quad \hat{K}_1 = \frac{2k_1}{B R_w^2}, \quad De_c = \frac{\lambda U_c}{R_w}, \quad X = \frac{U_c}{U_w}, \quad \frac{R_d}{R_w} = \delta > 1, \\ \lambda = \frac{1}{S_1^{\alpha-1}}, \quad Br = \frac{\eta_0 U_w^2}{k(\theta_d - \theta_w)}, \quad U_c = \frac{-B R_w^2}{8\eta_0},$$

Where De_c is Deborah number based on velocity scale U_c , $X = \frac{U_c}{U_w}$ has physical meaning of nondimensional pressure gradient and Br is the Brinkman number. From these dimensionless parameters using $De_c = \frac{U_c}{R_w S_1^{\alpha-1}}$, $C = \frac{-8 U_c \eta_0}{R_w^2}$, $k_1 = -4 U_c \eta_0 \hat{K}_1$, $S_1^{\alpha-1} = \frac{De_c R_w}{U_c}$, in **Eqn. 17**, after dropping the asterisks we obtain

$$\frac{dw}{dr} = \frac{-4 U_c}{R_w^2} r - \frac{4 U_c \hat{K}_1}{r} + (-4)^\alpha De_c R_w \eta_0^{\alpha-1} U_c^{\alpha-1} \hat{K}_1^\alpha r^{-\alpha} + \alpha (-4)^\alpha \frac{De_c}{R_w} \eta_0^{\alpha-1} U_c^{\alpha-1} \hat{K}_1^{\alpha-1} r^{2-\alpha} + \\ \frac{\alpha(\alpha-1)}{2!} (-4)^\alpha \frac{De_c}{R_w^3} \eta_0^{\alpha-1} U_c^{\alpha-1} \hat{K}_1^{\alpha-2} r^{4-\alpha} + \frac{\alpha(\alpha-1)(\alpha-2)}{3!} (-4)^\alpha \frac{De_c}{R_w^5} \eta_0^{\alpha-1} U_c^{\alpha-1} \hat{K}_1^{\alpha-3} r^{6-\alpha} + \dots \quad (18)$$

Taking $R_w = 1$, $U_w = 1$, then $U_c = X$, $R_d = \delta$, **Eqn. 18** yields

$$\frac{dw}{dr} = -4Xr - 4X\hat{K}_1 \frac{1}{r} + (-4)^\alpha De_c \eta_0^{\alpha-1} X^{\alpha-1} \sum_{k=0}^{\infty} \binom{\alpha}{k} \hat{K}_1^{(\alpha-k)} r^{(2k-\alpha)}, \quad (19)$$

Eqn.19 represent the axial velocity gradient in non-dimensional form and hence the boundary conditions given in **Eqn. 7**, reduces to

$$w(1) = 1, \quad w(\delta) = 0, \quad (20)$$

Now

The temperature distribution from **Eqn. 13** in non-dimensional form can be obtain as

$$k \left(\frac{d^2}{dr^2} + \frac{1}{r} \frac{d}{dr} \right) \theta + S_{rz} \frac{dw}{dr} = 0, \\ \text{using } S_{rz} = \left(\frac{B}{2} r + \frac{k_1}{r} \right), \text{ we obtain} \\ k \left(\frac{d^2}{dr^2} + \frac{1}{r} \frac{d}{dr} \right) \theta + \left(\frac{B}{2} r + \frac{k_1}{r} \right) \frac{dw}{dr} = 0, \\ \frac{k}{r} \frac{d}{dr} \left(r \frac{d\theta}{dr} \right) + \frac{1}{r} \left(\frac{B}{2} r^2 + k_1 \right) \frac{dw}{dr} = 0, \\ k \frac{d}{dr} \left(r \frac{d\theta}{dr} \right) + \left(\frac{B}{2} r^2 + k_1 \right) \frac{dw}{dr} = 0,$$

Using $B = \frac{-8 U_c \eta_0}{R_w^2}$, $k_1 = -4 U_c \eta_0 \hat{K}_1$, $R_w = 1$, $U_w = 1$, $U_c = X$, $\frac{\eta_0}{k} = Br$, in last equation, we obtain

$$\frac{d}{dr} \left(r \frac{d\theta}{dr} \right) - 4 Br X (r^2 + \hat{K}_1) \frac{dw}{dr} = 0, \quad (21)$$

With the boundary conditions given in **Eqn. 8**, reduces to $\theta(1) = 0$, $\theta(\delta) = 1$, (22)

Here we list the basic formulas related to the wire coating analysis for future use in our work.

The average velocity of fluid inside the die during coating process is given by

$$w_{ave} = \frac{2}{(R_d^2 - R_w^2)} \int_{R_w}^{R_d} r w(r) dr, \quad (23)$$

At some control surface downstream, the volume flow rate of coating is given by

$$Q = \pi U_w (R_c^2 - R_w^2), \quad (24)$$

Where R_c is the radius of the coated wire.

And to find the Volume flux, we use the formula given by the equation

$$Q = \int_{R_w}^{R_d} 2\pi r w(r) dr, \quad (25)$$

According, from both the **Eqs. 24-25** the thickness of the coated wire can be obtained as

$$R_c = \left(R_w^2 + \frac{2}{U_w} \int_{R_w}^{R_d} r w(r) dr \right)^{\frac{1}{2}}, \quad (26)$$

The force on the wire is computed by determining the shear stress at the wire surface, which is given by

$$S_{rz}|_{r=R_w} = \left(\frac{B}{2}r + \frac{k_1}{r} \right) \Big|_{r=R_w}, \quad (27)$$

The force exerted by the fluid on the total wire surface is given by the formula

$$F_w = 2\pi R_w L S_{rz}|_{r=R_w}, \quad (28)$$

Using $R_w = 1$, $U_w = 1$, then $U_c = X$, $R_d = \delta$, the system of Eqns. 23-28 reduces to

$$w_{ave} = \frac{w_{ave}(R_d^2 - R_w^2)}{2R_w^2 U_w}, \quad (29)$$

$$w_{ave} = \int_1^\delta r w(r) dr,$$

$$Q = \frac{Q}{2\pi R_w^2 U_w},$$

$$Q = \frac{\int_{R_w}^{R_d} 2\pi r w(r) dr}{2\pi R_w^2 U_w},$$

$$Q = \int_1^\delta r w(r) dr, \quad (30)$$

$$R_c = \frac{R_c}{R_w},$$

$$R_c = (1 + 2Q)^{\frac{1}{2}}, \quad (31)$$

$$S_{rz}|_{r=1} = \frac{R_w S_{rz}}{\eta_0 U_c} \Big|_{r=1},$$

$$S_{rz}|_{r=1} = -4(1 + \hat{K}_1), \quad (32)$$

$$F_w|_{r=1} = \frac{F_w}{2\pi \eta_0 L U_c} \Big|_{r=1},$$

$$F_w|_{r=1} = -4(1 + \hat{K}_1),$$

$$F_w|_{r=1} = S_{rz}|_{r=1}, \quad (33)$$

The stress component S_{rz} in dimensionless form is given by

$$S_{rz} = \frac{R_w S_{rz}}{\eta_0 U_c},$$

$$S_{rz} = \frac{1}{\eta_0 U_c} \left(\frac{B}{2}r + \frac{k_1}{r} \right),$$

$$S_{rz} = -4 \left(r + \frac{\hat{K}_1}{r} \right) \quad (34)$$

Now

Integrating Eqn. 19 with respect to r, we obtain

$$w(r) = -2Xr^2 - 4X\hat{K}_1 \ln(r) + (-4)^\alpha D_{\theta_c} \eta_0^{\alpha-1} X^{\alpha-1} \sum_{k=0}^{\infty} \binom{\alpha}{k} \hat{K}_1^{(\alpha-k)} \frac{r^{(2k-\alpha+1)}}{(2k-\alpha+1)} + k_2, \quad (35)$$

Where k_2 is another constant of integration to be determined.

The expression of velocity profile given in Eqn. 35 involves two constants $\hat{K}_1$ and k_2 that can be determined explicitly by using the boundary conditions, $w(1) = 1$, $w(\delta) = 0$, also this equation contains an infinite series for which we want to discuss a special case by taking $\alpha = 0$ in order to find the constant $\hat{K}_1$ and k_2 .

Special Case: when $\alpha = 2$, in Eqn. 35, we obtain

Inserting the boundary conditions from **Eqn. 20** into **Eqn. 35**, we end up with a quadratic equation,

$$\dot{K}_1^2 + Q_0 \dot{K}_1 + Q_1 = 0, \quad (36)$$

with coefficients,

$$Q_0 = \frac{S_1}{S_2}, \quad Q_1 = \frac{S_0}{S_2},$$

$$S_0 = \frac{16}{3} D_{\theta_c} \eta_0 X (1 - \delta^3) + 2X(\delta^2 - 1) - 1,$$

$$S_1 = 4X(\ln(\delta) + 8D_{\theta_c} \eta_0 (1 - \delta)),$$

$$S_2 = 16D_{\theta_c} \eta_0 X \left(\frac{1}{\delta} - 1\right),$$

The real root to the quadratic **Eqn. 36** can be obtain by taking the discriminant $Q_0^2 - 4Q_1 = 0$.

we get,

$$\dot{K}_1 = \frac{X(\ln \delta + 8D_{\theta_c} \eta_0 (1 - \delta))}{8D_{\theta_c} \eta_0 X \left(1 - \frac{1}{\delta}\right)},$$

The second constant of integration k_2 appeared in the axial velocity profile can be determined by the following method.

If both boundaries are stationary or moving then use of any boundary condition gives the second constant k_2 on the other hand if one boundary is stationary and other is moving with constant speed, then k_2 can be determined from the condition on the stationary boundary, otherwise the solution will not satisfy both the boundary conditions. Thus we determine the second constant k_2 from **Eqn. 35** by setting the no-slip condition at the die wall as,

$$k_2 = 2\delta^2 X + 4\dot{K}_1 X \ln(\delta) - \frac{16}{3} D_{\theta_c} \eta_0 X \delta^3 - 32D_{\theta_c} \eta_0 X \dot{K}_1 \delta + \frac{16D_{\theta_c} \eta_0 X \dot{K}_1^2}{\delta},$$

The velocity profile in dimensionless form

$$w(r) = -2Xr^2 - 4X\dot{K}_1 \ln(r) + 16D_{\theta_c} \eta_0 X \left(\frac{r^3}{3} + 2\dot{K}_1 r - \frac{\dot{K}_1^2}{r}\right) + k_2, \quad (37)$$

Volume flow rate $Q^* = \int_1^\delta r w(r) dr$ in dimensionless form is

$$Q^* = -\frac{X}{2}(\delta^4 - 1) - X\dot{K}_1(2\delta^2 \ln(\delta) - \delta^2 + 1) + 16D_{\theta_c} \eta_0 X \left(\frac{1}{15}(\delta^5 - 1) + \frac{2}{3}\dot{K}_1(\delta^3 - 1) - \dot{K}_1^2(\delta - 1)\right) + \frac{k_2}{2}(\delta^2 - 1) \quad (38)$$

Thickness of the coated wire $R_c^* = (1 + 2Q^*)^{\frac{1}{2}}$ in dimensionless form is

$$R_c^* = \left(1 - X(\delta^4 - 1) - 2X\dot{K}_1(2\delta^2 \ln(\delta) - \delta^2 + 1) + 32D_{\theta_c} \eta_0 X \left(\frac{1}{15}(\delta^5 - 1) + \frac{2}{3}\dot{K}_1(\delta^3 - 1) - \dot{K}_1^2(\delta - 1)\right) + k_2(\delta^2 - 1)\right)^{\frac{1}{2}}, \quad (39)$$

The force on the total wire in dimensionless form is

$$F_w^* = -4(1 + \dot{K}_1), \quad (40)$$

This expression shows an important result that the force on the wire is constant throughout the domain and which depends upon the parameters involved in constant $\dot{K}_1$.

To find Temperature distribution we use the value of $\frac{dw}{dr}$ from **Eqn. 19** in **Eqn. 21** with boundary conditions,

$\Theta(1) = 0$, $\Theta(\delta) = 1$, we obtain

$$\begin{aligned} & \frac{d}{dr} \left(r \frac{d\Theta}{dr} \right) - 4 Br X (r^2 + \dot{K}_1) \left(-4 r X - 4 \dot{K}_1 X \frac{1}{r} + 16 De_c \eta_0 X (r^2 + 2 \dot{K}_1 - \dot{K}_1^2 \frac{1}{r^2}) \right) = 0, \\ & \Theta(r) = -4 Br X \left(\frac{X}{4} (r^4 - 1) + 2 \dot{K}_1 X (r^2 - 1) + 2 \dot{K}_1^2 X \ln(r)^2 - 16 De_c \eta_0 X \left(3 \dot{K}_1^2 (r - 1) + \right. \right. \\ & \left. \left. \dot{K}_1^3 \left(\frac{1}{r} - 1 \right) + \frac{1}{25} (r^5 - 1) + \frac{\dot{K}_1}{3} (r^3 - 1) \right) \right) + 4 Br X \frac{\ln(r)}{\ln(\delta)} \left(\frac{X}{4} (\delta^4 - 1) + 2 \dot{K}_1 X (\delta^2 + \ln(\delta) - 1) - \right. \\ & \left. 16 De_c \eta_0 X \left(3 \dot{K}_1^2 (\delta - 1) + \frac{\dot{K}_1^3}{9 \delta} (\delta^4 - 9 \delta + 9) + \frac{1}{25} (\delta^5 - 1) + \frac{\dot{K}_1}{9} (2 \delta^2 - 3) \right) \right) + \frac{\ln(r)}{\ln(\delta)} \end{aligned} \quad (41)$$

4. The results and discussion. Eqn. 19 and Eqn. 21 along with the boundary conditions given in Eqn. 20 and Eqn. 22 were solved for velocity field and temperature distribution by considering a special case for the parameter α . Moreover, the volume flow rate, average velocity, thickness of the coated wire, shear stress and force on total wire are derived using velocity field. All the results are observed physically through Figs. (2)-(10). The effect of dimensionless parameters De_c , X and Brinkman number Br are discussed. Figs. (2)-(3), represent the velocity profiles as a function of 'r' for several values of dimensionless numbers X and De_c . In Figs. (2) we varied the ratio between the pressure drop and speed of wire $\left(X = \frac{U_c}{U_w} \right)$, that is, $X = 1.1, 1.3, 1.7, 2.2$, fixing $\delta = 2, De_c = 0.01, \eta_0 = 0.7$, which shows that the rise in pressure gradient increases the speed of flow. Fig. (3) is sketched for $De_c = 3, 5, 8, 12$, fixing $\delta = 2, \eta_0 = 0.7, X = 0.5$. It is obvious from the Figs. (2)-(3) that velocity decreases with increase in dimensionless numbers X and decreases for De_c . Fig. (4) is plotted for variation of shear stress for different values of De_c . It is to be noted that an increase in the parameter De_c , shear stress increases which is insignificant for small and large values of De_c . The Fig. (5) is plotted for variation of volume flow rate for different values of De_c fixing $X = 0.5, \eta_0 = 0.7$. Here it is observed that the flow rate decreases for increasing values of De_c . The thickness of the coated wire is plotted in the Fig. (6) for different values of De_c fixing $X = 0.5, \eta_0 = 0.7$, that also show decrease in the thickness. The dimensionless force on the total wire versus δ is sketched in the Fig. (7) for various values of De_c fixing $X = 0.5, \eta_0 = 0.7$. It is to be noted that the graph show increase in the force on total wire as δ increases. In Figs. (8)-(10), we plotted the dimensionless temperature profile $\Theta(r)$ versus δ with selected sets of parameters Br, De_c , and X . It is obvious from the Figs. (8)-(10) that temperature increases as these parameters increases, It is also observed that temperature increases very rapidly for the increasing values of the parameters X and De_c . Thus we concluded that the temperature distribution increases as the dimensionless numbers Br, De_c and X increases.

Figure 2: Dimensionless velocity profiles for different values of $X = 1.1, 1.3, 1.7, 2.5, 2.2$ at fixed values of $\delta = 2, De_c = 0.01, \eta_0 = 0.7$.

Figure 3: Dimensionless velocity profiles for different values of $De_c = 3, 5, 8, 12$, at fixed values of $\delta = 2, X = 0.5, \eta_0 = 0.7$.

Figure 4: Dimensionless shear stress profiles for different values of $De_c = 1, 10, 30, 50$, at fixed values of $\delta = 2, X = 0.5, \eta_0 = 0.7$.

Figure 5: Dimensionless volume flow rates versus ratio of the radii for different values of $De_c = 0.01, 0.03, 0.07, 0.09$, at fixed values of $X = 0.5, \eta_0 = 0.7$.

Figure 6: Thickness of the coated wire versus ratio of the radii for different values of $De_c = 0.01, 0.03, 0.07, 0.09$, at fixed values of $X = 0.5$, $\eta_0 = 0.7$.

Figure 7: Force on the surface of the total wire for different values of $De_c = 2, 7, 12, 16$, at fixed values of $X = 0.5$, $\eta_0 = 0.7$.

Figure 8: Dimensionless temperature distribution for different values of Brinkman number $Br = 0.1, 3, 7, 12$, at fixed values of $X = 0.5, \delta = 2, \eta_0 = 0.7, De_c = 3$.

Figure 9: Dimensionless temperature distribution for different values of $De_c = 0.1, 2, 5, 12$, at fixed values of $\delta = 2, X = 0.5, \eta_0 = 0.7, Br = 5$.

Figure 10: Dimensionless temperature distribution for different values of $X = 0.1, 0.4, 0.8, 1.3$, at fixed values of $\delta = 2, De_c = 3, \eta_0 = 0.7, Br = 5$.

5. Conclusion. Analytical solutions are derived for the flow of nonlinear Ellis fluids model in wire coating analysis by taking a special case for $\alpha = 2$, after the application of Binomial series technique. Expressions are presented for the radial variation of the axial velocity and the temperature distribution. In engineering point of view, some results are also derived like volume flow rate, average velocity, shear stress, thickness of coated wire and force on the total wire. It was found that the axial velocity depends upon the dimensionless parameters X and De_c . Moreover, the velocity profile increases with different (increasing) values of the parameter X and decreases with De_c . It is to be noted that the shear stress, volume flow rate and thickness of the coated wire decreases for increasing values of De_c . The dimensionless force on the total wire shows increase as De_c increases. Finally, the fluid temperature depends upon the dimensionless numbers Br, X, De_c and it increases with increasing values of these parameters.

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