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Efficient Neighbor Designs Weakly Balanced in Circular Blocks of Three Different Sizes

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Abstract Minimal circular weakly balanced neighbor designs (MCWBNDs) are efficient to control neighbor effects for v even. MCWBNDs-II are designs where $\frac{3v}{2}$ unordered pairs of different treatments occur twice but remaining pairs occur once as neighbors, are not available for $m \pmod 4) \equiv 0$ & 3 in blocks of three different sizes with $m = \frac{(v-2)}{2}$ and v even. Here, some new generators are presented to develop cyclic shifts to get MCWBNDs-II in blocks of three different sizes.

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1 Introduction

Balanced neighbor designs (BNDs) are used to control neighbor effects in experiments where neighboring unit treatments influence the effect of current unit treatment. Bias due to neighbor effects is minimized with the use of balanced neighbor designs (BNDs) or partially BNDs, see Azais (1987) [3], Langton (1990) [13], Azais et al. (1993) [4], Kunert (2000) [12] and Tomar et al. (2005) [27]. If every treatment appears once with all other treatments as neighbor then design is called minimal neighbor balanced. If each treatment appears with other treatments as neighbor either once or twice then designs are called weakly balanced neighbor design (WBNDs). Minimal circular WBNDs (MCWBNDs-II) are the designs in which $\frac{3v}{2}$ of the unordered pairs appear twice while the remaining ones appear one time as neighbors.

Williams (1952) [28] suggested neighbor balance designs in linear blocks. In circular blocks, neighbor designs were used in virus research by Rees (1967) [22]. Hwang (1973) [7] and Azaiz et al. (1993) [4] constructed BNDs for different configurations of k (block size) and v (number of treatments). Using cyclic shifts, Iqbal et al. (2009) [9], Akhtar et al. (2010) [2], Ahmed and Akhtar (2011) [1] constructed some series of BNDs. Shehzad et al. (2011) [26] constructed minimal circular BNDs (MCBNDs) for some cases. Khalid et al. (2019) [11] and Shahid et al. (2019) [25] presented some generators for MBNDs and MPBNDs in linear blocks, respectively. Rasheed et al. (2019) [20] developed catalogues of some MCWBNDs. Nadeem et al. (2021) [15] constructed some classes of MCGNDs. Salam et al. (2021) [23] gave some new constructions of MCBNDs. MCWBNDs-II are preferred for the combinations of v and k's where MCBNDs and MCWBNDs-I cannot be generated. MCWBNDs-I are the designs in which $\frac{v}{2}$ of the unordered pairs appear twice while the remaining ones appear one time as neighbors. Rasheed et al. (2022) [21] have developed generators to get MCWBNDs-II for m (mod 4) $\equiv$ 1 & 2, where $m = \frac{(v-2)}{2}$ with v even. Generators for these designs have not been developed for m (mod 4) $\equiv$ 0 & 3. Noreen et al. (2022) [17] developed an R-coded algorithm based on Rule I to generate MCPBRMDs. Shabbir et al. (2023) [24] developed an R-coded algorithm based on Rule I to generate GN2-designs. Hassan et al. (2023) [6] developed Quasi rees neighbor designs which can be converted into other useful classes. Fardos et al. (2023) [5] presented catalogues of some important classes of MCBNDs. Noreen et al. (2023 a, b) [16, 18] constructed MCNSBNDs and MCWBNDs, respectively.

In this article, generators are developed to get MCWBNDs-II when m (mod 4) $\equiv$ 0 & 3, in blocks of three different sizes. MCWBNDs-II are efficient and economical to control neighbor effects for $v = 2ik_1 + 2k_2 + 2k_3 - 2$, $v = 2ik_1 + 2k_2 + 4k_3 - 2$, $v = 2ik_1 + 4k_2 + k_3 - 2$ and $v = 2ik_1 + 4k_2 + 4k_3 - 2$. Efficiency of neighbor effects for each design is also evaluated.

2 Methodology

Iqbal (1991) [8] developed cyclic shifts method to construct several types of designs. Its Rule I is described here for MCWBNDs-II. Let $S_j = [q_{j1}, q_{j2}, \cdots, q_{j(k-1)}]$ be i sets of shifts, where $j = 1, 2, \cdots, i$, $1 \leq q_{ju} \leq v-1$, $u = 1, 2, \cdots, k-1$. If each of $1, 2, \cdots, v-1$ appears exactly once except $\frac{v}{2}$ and two more values which appear twice in S^* then design is MCWBND-II with S^* includes:

(i) Every element of all S_j with sum (mod v) of each S_j.
(ii) Complements of all values in (i) with 'v-a' as complement of 'a'.

Example 1. $[3, 7, 10, 11, 16] + [4, 6, 12] + [9, 10]$ *produce following MCWBND-II for* $v = 24$, $k_1 = 6$, $k_2 = 4$, $k_3 = 3$.

Table 1. Blocks generated from [3, 7, 10, 11, 16]

Blocks																							
1	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**	**13**	**14**	**15**	**16**	**17**	**18**	**19**	**20**	**21**	**22**	**23**	**24**
0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23
3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	0	1	2
10	11	12	13	14	15	16	17	18	19	20	21	22	23	0	1	2	3	4	5	6	7	8	9
20	21	22	23	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19
7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	0	1	2	3	4	5	6
23	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22

Table 1, 2 & 3 jointly produce MCWBND-II for $v = 24$, $k_1 = 6$, $k_2 = 4$, $k_3 = 3$. In this design, 36 pairs occur twice while the remaining 276 occur once as neighbor.

Table 2. Blocks generated from [4, 6, 12]

Blocks																							
25	**26**	**27**	**28**	**29**	**30**	**31**	**32**	**33**	**34**	**35**	**36**	**37**	**38**	**39**	**40**	**41**	**42**	**43**	**44**	**45**	**46**	**47**	**48**
0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23
4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	0	1	2	3
10	11	12	13	14	15	16	17	18	19	20	21	22	23	0	1	2	3	4	5	6	7	8	9
22	23	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21

Table 3. Blocks generated from [9, 10]

Blocks																							
49	**50**	**51**	**52**	**53**	**54**	**55**	**56**	**57**	**58**	**59**	**60**	**61**	**62**	**63**	**64**	**65**	**66**	**65**	**68**	**69**	**70**	**71**	**72**
0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23
9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	0	1	2	3	4	5	6	7	8
19	20	21	22	23	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18

3 Efficiency of Neighbor Effects

Following model is considered assuming the left neighbor and right neighbor effects same.

$$y_{ij} = \mu + \beta_i + \tau_{d(i,j)} + \lambda_{d(i,j-1 \text{ or } j+1)} + \epsilon_{ij} \tag{1}$$

According to James and Wilkinson (1971) [10] and Pearce et al. (1974) [19], harmonic mean of non-zero Eigen values of the respective information matrix are regarded as the efficiency factor for neighbor effect (residual effect). Design possessing the high value of E_n will be efficient to control neighbor effects.

4 Constructors for MCWBNDs-II

In this construction, v is even and $m = \frac{(v-2)}{2}$. Following constructors are developed by Mehmood et al. (2022) [14].

Constructor 1. *Sets of shifts for MCWBNDs-II can be obtained from* $A = [1, 2, \cdots, m-1, m-1, m+1, m+2]$ *for* $m \pmod 4 \equiv 0$.

Constructor 2. *Sets of shifts for MCWBNDs-II can be obtained from* $B = [1, 2, \cdots, \frac{(3m-5)}{4}, \frac{(3m+3)}{4}, \frac{(3m+7)}{4}, \cdots, m-1, m, m+1, m-1, \frac{(5m+9)}{4}]$ *for* $m \pmod 4 \equiv 3$.

4.1 Procedure to Obtain Sets of shifts for MCWBNDs-II in Three Different Blocks Sizes

Sets of shifts to generate MCWBNDs-II are obtained as:

- Divide elements of constructor A or B into i classes of k_1 values and (i) one each of k_2 and k_3 values for $v = 2ik_1 + 2k_2 + 2k_3 - 2$, (ii) one class of k_2 and two of k_3 values for $v = 2ik_1 + 2k_2 + 4k_3 - 2$, (iii) two of k_2 and one of k_3 values for $v = 2ik_1 + 4k_2 + 2k_3 - 2$, (iv) two each of k_2 and k_3 values for $v = 2ik_1 + 4k_2 + 4k_3 - 2$, such that their sum is divisible by v.
- Delete anyone element from each class.

5 Generators for MCWBNDs-II in Three Different Blocks Sizes for m (mod 4) $\equiv$ 0

Here, some generators are developed under the logic of Rule I to obtain sets of shifts for MCWBNDs-II in blocks of three different sizes for m (mod 4) $\equiv$ 0.

5.1 For $k_2 = k_1 - 1$ and $k_3 = k_1 - 2$

Generator 1. *MCWBNDs-II for* $v = 2ik_1 + 2k_2 + 2k_3 - 2$ *with* $k_2 = k_1 - 1$, $k_3 = k_1 - 2$ *can be obtained if:*

- k_1 *(mod 4)* $\equiv 1$, *i (mod 4)* $\equiv 3$.
- k_1 *(mod 4)* $\equiv 3$, *i (mod 4)* $\equiv 1$.

Example 2. *MCWBNDs-II obtained through Generator 1. See Table 4*

Table 4. MCWBNDs-II obtained through Generator 1.

v	k_1	k_2	k_3	i	Sets of Shifts	E_n
42	5	4	3	i (mod 4) $\equiv$ 3	[4,5,9,21], [7,8,10,15], [16,17,18,19], [11,12,13], [19,22]	0.786
34	7	6	5	i (mod 4) $\equiv$ 1	[2,4,5,6,7,9], [10,11,12,13,14], [15,15,17,18]	0.854

Generator 2. *MCWBNDs-II for* $v = 2ik_1 + 2k_2 + 4k_3 - 2$ *with* $k_2 = k_1 - 1$, $k_3 = k_1 - 2$ *can be obtained if:*

- k_1 (mod 4) $\equiv$ 1, i (mod 4) $\equiv$ 0.
- k_1 (mod 4) $\equiv$ 3, i (mod 4) $\equiv$ 2.

Example 3. *MCWBNDs-II obtained through Generator 2. See Table 5.*

Table 5. MCWBNDs-II obtained through Generator 2.

v	k_1	k_2	k_3	i	Sets of Shifts	E_n
58	5	4	3	i (mod 4) $\equiv$ 0	[20,22,26,30], [8,9,10,25], [11,13,14,15], [3,12,19,23], [16,17,21], [24,27], [27,29]	0.774
58	7	6	5	i (mod 4) $\equiv$ 2	[2,4,5,6,14,26],[11,12,13,21,25,24],[17,18,20,22,23], [8,9,15,19],[27,27,29,30]	0.845

Generator 3. *MCWBNDs-II for* $v = 2ik_1 + 4k_2 + 2k_3 - 2$ *with* $k_2 = k_1 - 1$, $k_3 = k_1 - 2$ *can be obtained if:*

- $k_1 = 4l + 2$, i even.
- k_1 (odd) > 3, i (mod 4) $\equiv$ 3.

Example 4. *MCWBNDs-II obtained through Generator 3. See Table 6.*

Generator 4. *MCWBNDs-II for* $v = 2ik_1 + 4k_2 + 4k_3 - 2$ *with* $k_2 = k_1 - 1$, $k_3 = k_1 - 2$ *can be obtained if:*

- $k_1 = 4l$, i integer & $l > 1$.

Table 6. MCWBNDs-II obtained through Generator 3.

v	k_1	k_2	k_3	i	Sets of Shifts	E_n
50	6	5	4	even	[4,5,6,10,23], [7,8,9,11,12], [16,18,25,26], [19,20,21,23], [13,14,22]	0.821
50	5	4	3	$i \pmod 4 \equiv 3$	[3,4,16,26], [20,21,22,23], [7,9,13,15], [8,18,19], [11,12,17], [23,25]	0.781

- $k_1 = 4l + 2$, i even.
- k_1 (odd) > 3, $i \pmod 4 \equiv 0$.

Example 5. *MCWBNDs-II obtained through Generator 4. See Table 7.*

Table 7. MCWBNDs-II obtained through Generator 4.

v	k_1	k_2	k_3	i	Sets of Shifts	E_n
66	8	7	6	integer	[2,3,5,6,7,8,34], [12,13,16,23,24,33], [17,18,19,20,21,22], [14,25,26,27,30], [9,29,31,28,31]	0.864
58	6	5	4	even	[3,4,5,14,30], [15,20,22,23,25], [18,26,27,29], [8,9,13,21], [12,17,19], [6,24,27]	0.811
66	5	4	3	$i \pmod 4 \equiv 0$	[3,4,24,34], [8,12,14,25], [11,13,15,18], [26,27,28,30], [16,22,23], [17,19,20], [29,31], [31,33]	0.773

5.2 For $k_2 = k_1 - 1$ and $k_3 = k_1 - 3$

Generator 5. *MCWBNDs-II for $v = 2ik_1 + 2k_2 + 2k_3 - 2$ with $k_2 = k_1 - 1$, $k_3 = k_1 - 3$ can be obtained if:*

- $k_1 = 4l + 2$, l integer, i odd.
- k_1 (odd) > 5, $i \pmod 4 \equiv 0$.

Example 6. *MCWBNDs-II obtained through Generator 5. See Table 8.*

Table 8. MCWBNDs-II obtained through Generator 5.

v	k_1	k_2	k_3	i	Sets of Shifts	E_n
26	6	5	3	Odd	[5,9,10,11,13], [3,6,7,8], [11,14]	0.813
74	7	6	4	$i \pmod 4 \equiv 0$	[7,18,21,22,37,38], [13,14,19,31,28,32], [4,6,9,16,17,20], [15,23,24,25,26,27], [12,29,30,33,34], [3,35,35]	0.851

Generator 6. *MCWBNDs-II for $v = 2ik_1 + 2k_2 + 4k_3 - 2$ with $k_2 = k_1 - 1$, $k_3 = k_1 - 3$ can be obtained if:*

- $k_1 \pmod 4 \equiv 1$, $i \pmod 4 \equiv 2$.
- $k_1 \pmod 4 \equiv 3$, $i \pmod 4 \equiv 0$.

Example 7. *MCWBNDs-II obtained through Generator 6. See Table 9.*

Table 9. MCWBNDs-II obtained through Generator 6.

v	k_1	k_2	k_3	i	Sets of Shifts	E_n
74	9	8	6	$i \pmod 4 \equiv 2$	[2,3,5,6,7,8,9,33], [12,13,14,15,16,17,18,32], [22,27,28,30,31], [23,24,25,26,29,37,38], [19,21,35,34,35]	0.878
82	7	6	4	$i \pmod 4 \equiv 0$	[7,17,22,34,37,42],[8,9,10,12,18,19], [29,31,32,33,36], [14,16,20,21,39,41],[23,24,25,26,27,28],[15,30,35],[4,38,39]	0.842

Generator 7. *MCWBNDs-II for* $v = 2ik_1 + 4k_2 + 2k_3 - 2$ *with* $k_2 = k_1 - 1$, $k_3 = k_1 - 3$ *can be obtained if*

- $k_1 \pmod 4 \equiv 3$ & $i \pmod 4 \equiv 0$.

Example 8. *MCWBNDs-II obtained through Generator 7. See Table 10.*

Table 10. MCWBNDs-II obtained through Generator 7.

v	k_1	k_2	k_3	i	Sets of Shifts	E_n
58	7	6	4	$i \pmod 4 \equiv 0$	[3,4,6,7,10,26], [11,12,13,14,27,30], [17,18,19,20,27], [21,22,23,24,25], [8,16,29]	0.845

Generator 8. *MCWBNDs-II for* $v = 2ik_1 + 4k_2 + 4k_3 - 2$ *with* $k_2 = k1 - 1$, $k_3 = k1 - 3$ *can be obtained if:*

- $k_1 = 4l + 2$, i odd.
- k_1 (odd) > 5, $i \pmod 4 \equiv 2$.

Example 9. *MCWBNDs-II obtained through Generator 8. See Table 11.*

Table 11. MCWBNDs-II obtained through Generator 8.

v	k_1	k_2	k_3	i	Sets of Shifts	E_n
42	6	5	3	integer	[2,3,6,8,22], [7,9,10,11], [15,16,18,21], [13,17], [4,19]	0.786
66	7	6	4	$i \pmod 4 \equiv 2$	[8,12,17,27,30,34], [6,9,10,11,13,14], [16,19,21,28,33], [20,22,23,24,25], [7,26,31], [5,29,31]	0.834

5.3 For $k_2 = k_1 - 2$ and $k_3 = k_1 - 3$

Generator 9. *MCWBNDs-II for* $v = 2ik_1 + 2k_2 + 2k_3 - 2$ *with* $k_2 = k_1 - 2$, $k_3 = k_1 - 3$ *can be obtained if:*

- $k_1 \pmod 4 \equiv 1$, $i \pmod 4 \equiv 1$.
- $k_1 \pmod 4 \equiv 3$, $i \pmod 4 \equiv 3$.

Example 10. *MCWBNDs-II obtained through Generator 9. See Table 12.*

Generator 10. *MCWBNDs-II for* $v = 2ik_1 + 2k_2 + 4k_3 - 2$ *with* $k_2 = k_1 - 2$, $k_3 = k_1 - 3$ *can be obtained if:*

Table 12. MCWBNDs-II obtained through Generator 9.

v	k_1	k_2	k_3	i	Sets of Shifts	E_n
80	9	7	6	$i \pmod 4 \equiv 1$	[4,5,6,7,22,35,38,40], [10,15,16,17,21,23,24,25], [12,13,14,18,19,20,26,27], [30,31,32,33,51,34], [8,36,37,38,39]	0.884
58	7	5	4	$i \pmod 4 \equiv 3$	[3,4,5,6,12,27], [10,11,13,16,30,29], [9,17,19,20,21,22], [23,24,25,26], [14,15,27]	0.845

Table 13. MCWBNDs-II obtained through Generator 10.

v	k_1	k_2	k_3	i	Sets of Shifts	E_n
42	6	4	3	even	[4,5,6,8,16], [10,11,14,19,21], [12,13,15], [17,18], [19,22]	0.785
66	7	5	4	$i \pmod 4 \equiv 3$	[3,4,6,7,10,34], [11,12,13,28,30,33], [17,18,19,20,21,22], [,24,25,29,31], [14,16,27], [8,26,31]	0.833

- $k_1 = 4l + 2$, i even & l integer.
- k_1 (odd) > 5, $i \pmod 4 \equiv 3$.

Example 11. *MCWBNDs-II obtained through Generator 10. See Table 13.*

Generator 11. *MCWBNDs-II for $v = 2ik_1 + 4k_2 + 2k_3 - 2$ with $k_2 = k_1 - 2$, $k_3 = k_1 - 3$ can be obtained if:*

- $k_1 \pmod 4 \equiv 1$, $i \pmod 4 \equiv 2$.
- $k_1 \pmod 4 \equiv 3$, $i \pmod 4 \equiv 0$.

Example 12. *MCWBNDs-II obtained through Generator 11. See Table 14.*

Table 14. MCWBNDs-II obtained through Generator 11.

v	k_1	k_2	k_3	i	Sets of Shifts	E_n
74	9	7	6	$i \pmod 4 \equiv 2$	[12,16,17,30,32,34,35,35], [5,9,14,18,20,22,24,33], [10,15,21,28,31,37], [4,23,25,27,29,38], [7,8,13,19,26]	0.742
82	7	5	4	$i \pmod 4 \equiv 0$	[4,5,6,7,16,42], [8,9,10,11,13,28], [15,18,19,21,36,41], [22,23,24,25,26,27], [31,32,33,39], [34,35,37,38], [12,30,39]	0.842

Generator 12. *MCWBNDs-II for $v = 2ik_1 + 4k_2 + 4k_3 - 2$ with $k_2 = k_1 - 2$, $k_3 = k_1 - 3$ can be obtained if:*

- $k_1 = 4l$, i integer & $l > 1$.
- $k_1 = 4l + 2$, i even.
- k_1 (odd) > 5, $i \pmod 4 \equiv 0$.

Example 13. *MCWBNDs-II obtained through Generator 12. See Table 15.*

Table 15. MCWBNDs-II obtained through Generator 12.

v	k_1	k_2	k_3	i	Sets of Shifts	E_n
58	8	6	5	integer	[6,7,14,15,19,20,30], [9,10,11,12,13], [2,4,16,17,18], [22,23,24,26], [25,27,27,29]	0.845
50	6	4	3	even	[3,5,6,9,26], [12,16,18,20,23], [13,14,15], [10,17,19], [21,22], [23,25]	0.778
90	7	5	4	$i \pmod 4 \equiv 0$	[3,5,6,7,21,46], [9,10,11,12,14,26], [13,18,19,41,40,45], [22,23,24,25,27,39], [,32,33,42,43], [35,36,37,38],[17,28,29], [15,31,43]	0.833

5.4 For $k_2 = k_1 - 1$ and $k_3 = 3, 4, 5$

Generator 13. *MCWBNDs-II for* $v = 2ik_1 + 2k_2 + 2k_3 - 2$ *with* $k_2 = k_1 - 1$, $k_3 = 3$ *can be obtained if:*

- $k_1 = 4l$, i integer & $l > 1$.
- $k_1 = 4l + 2$, i odd & l integer.
- k_1 (odd) > 3, $i \pmod 4 \equiv 3$.

Example 14. *MCWBNDs-II obtained through Generator 13. See Table 16.*

Table 16. MCWBNDs-II obtained through Generator 13.

v	k_1	k_2	k_3	i	Sets of Shifts	E_n
34	8	7	3	integer	[5,6,7,8,10,11,17], [3,9,12,14,13,15], [15,18]	0.855
26	6	5	3	odd	[5,9,10,11,13], [3,6,7,8],[11,14]	0.813
42	5	4	3	$i \pmod 4 \equiv 3$	[4,5,13,17], [15,16,18,21], [8,9,11,12], [7,10,19], [19,22]	0.788

Generator 14. *MCWBNDs-II for* $v = 2ik_1 + 2k_2 + 2k_3 - 2$ *with* $k_2 = k_1 - 1$, $k_3 = 4$ *can be obtained if:*

- $k_1 \pmod 4 \equiv 1$, $i \pmod 4 \equiv 2$.
- $k_1 \pmod 4 \equiv 3$, $i \pmod 4 \equiv 0$.

Example 15. *MCWBNDs-II obtained through Generator 14. See Table 17.*

Table 17. MCWBNDs-II obtained through Generator 14.

v	k_1	k_2	k_3	i	Sets of Shifts	E_n
58	9	8	4	$i \pmod 4 \equiv 2$	[6,7,8,9,10,14,27,30], [4,11,13,15,16,17,18,19], [20,21,22,23,24,25,27], [2,26,29]	0.880
74	7	6	4	$i \pmod 4 \equiv 0$	[6,7,28,32,33,38], [9,10,11,12,13,14], [17,18,19,21,22,35], [15,23,24,25,26,27], [20,29,30,31,35], [2,34,37]	0.851

Generator 15. *MCWBNDs-II for* $v = 2ik_1 + 2k_2 + 2k_3 - 2$ *with* $k_2 = k_1 - 1$, $k_3 = 5$ *can be obtained if:*

- k_1 (odd) > 5, $i \pmod 4 \equiv 1$.
- $k_1 = 4l + 2$, i even.

Example 16. *MCWBNDs-II obtained through Generator 15. See Table 18.*

Table 18. MCWBNDs-II obtained through Generator 15.

v	k_1	k_2	k_3	i	Sets of Shifts	E_n
34	7	6	5	$i \pmod 4 \equiv 1$	[2,3,5,6,7,10], [9,11,12,13,15], [14,15,17,18]	0.852
66	10	9	5	even	[2,3,5,6,7,8,9,10,15], [14,17,18,19,20,21,23,22,31], [12,16,24,25,26,27,28,29], [30,31,33,34]	0.789

5.5 For $k_2 = k_1 - 2$ and k3 = 3, 4, 5

Generator 16. *MCWBNDs-II for $v = 2ik_1 + 2k_2 + 2k_3 - 2$ with $k_2 = k_1 - 2$, $k_3 = 3$ can be obtained if:*

- $k_1 \pmod 4 \equiv 1$, $i \pmod 4 \equiv 0$.
- $k_1 \pmod 4 \equiv 3$, $i \pmod 4 \equiv 2$.

Example 17. *MCWBNDs-II obtained through Generator 16. See Table 19.*

Table 19. MCWBNDs-II obtained through Generator 16.

v	k_1	k_2	k_3	i	Sets of Shifts	E_n
90	9	7	3	$i \pmod 4 \equiv 0$	[4,5,6,7,8,9,13,37], [14,15,16,17,18,23,28,46], [12,19,20,21,22,24,25,26], [29,30,32,31,33,34,35,36], [38,39,40,41,42,43], [43,45]	0.878
42	7	5	3	$i \pmod 4 \equiv 2$	[3,4,5,6,7,15], [9,10,11,13,12,21], [16,17,18,19], [19,22]	0.834

Generator 17. *MCWBNDs-II for $v = 2ik_1 + 2k_2 + 2k_3 - 2$ with $k_2 = k_1 - 2$, $k_3 = 4$ can be obtained if:*

- $k_1 = 4l$, i integer & $l > 1$.
- $k_1 = 4l + 2$, i odd.
- k_1 (odd) > 5 and $i \pmod 4 \equiv 3$.

Example 18. *MCWBNDs-II obtained through Generator 17. See Table 20.*

Table 20. MCWBNDs-II obtained through Generator 17.

v	k_1	k_2	k_3	i	Sets of Shifts	E_n
34	8	6	4	integer	[4,5,6,8,9,15,18], [10,11,12,13,15], [2,14,17]	0.854
42	8	6	4	odd	[19,5,6,7,8,9,10,13,4], [12,14,15,16,17,19,22], [2,18,21]	0.849
58	7	5	4	$i \pmod 4 \equiv 3$	[6,13,16,25,26,27], [11,12,17,21,22,24], [2,4,7,10,15,19], [20,23,29,30], [8,18,27]	0.846

Generator 18. *MCWBNDs-II for* $v = 2ik_1 + 2k_2 + 2k_3 - 2$ *with* $k_2 = k_1 - 2$, $k_3 = 5$ *can be obtained if*

- $k_1 \pmod 4 \equiv 1$, $i \pmod 4 \equiv 2$.

Example 19. *MCWBNDs-II obtained through Generator 18. See Table 21.*

Table 21. MCWBNDs-II obtained through Generator 18.

v	k_1	k_2	k_3	i	Sets of Shifts	E_n
58	9	7	5	$i \pmod 4 \equiv 0$	[6,7,8,9,10,14,27,30], [4,11,13,15,16,17,18,19], [2,20,21,22,23,27], [24,25,26,29]	0.879

6 Generators for MCWBNDs-II in Three Different Blocks Sizes for m (mod 4) ≡ 3

Here, some generators are developed under the logic of Rule I to obtain shifts for MCWBNDs-II in three different block sizes for $m \pmod 4 \equiv 3$.

6.1 For $k_2 = k_1 - 1$ and $k_3 = k_1 - 2$

Generator 19. *MCWBNDs-II for* $v = 2ik_1 + 2k_2 + 2k_3 - 2$ *with* $k_2 = k_1 - 1$, $k_3 = k_1 - 2$ *can be obtained if:*

- $k_1 = 4l$, i integer & $l > 1$.
- $k_1 = 4l + 2$, i even & $l > 1$.
- k_1 (odd) > 3, $i \pmod 4 \equiv 2$.

Example 20. *MCWBNDs-II obtained through Generator 19. See Table 22.*

Table 22. MCWBNDs-II obtained through Generator 19.

v	k_1	k_2	k_3	i	Sets of Shifts	E_n
40	8	7	6	integer	[2,3,4,6,7,8,9], [12,15,17,19,20,26], [10,13,16,18,18]	0.875
40	6	5	4	even	[3,4,5,7,19], [10,11,12,13,26], [16,17,18,20], [6,15,18]	0.826
32	5	4	3	$i \pmod 4 \equiv 2$	[4,5,6,14], [9,12,14,21], [7,10,13], [15,16]	0.783

Generator 20. *MCWBNDs-II for* $v = 2ik_1 + 2k_2 + 4k_3 - 2$ *with* $k_2 = k_1 - 1$, $k_3 = k_1 - 2$ *can be obtained if:*

- $k_1 = 4l + 2$, i even.
- k_1 (odd) > 3, $i \pmod 4 \equiv 2$.

Example 21. *MCWBNDs-II obtained through Generator 20. See Table 23.*

Generator 21. *MCWBNDs-II for* $v = 2ik_1 + 4k_2 + 2k_3 - 2$ *with* $k_2 = k_1 - 1$, $k_3 = k_1 - 2$ *can be obtained if:*

- $k_1 \pmod 4 \equiv 1$, $i \pmod 4 \equiv 2$.

Table 23. MCWBNDs-II obtained through Generator 20.

v	k_1	k_2	k_3	i	Sets of Shifts	E_n
48	6	5	4	even	[9,14,21,23,24], [4,7,11,12,13], [16,19,20,31], [8,15,22], [6,18,22]	0.813
48	5	4	3	$i \pmod 4 \equiv 2$	[4,6,14,22], [7,8,10,18], [13,19,21,31], [9,16,20], [15,22], [23,24]	0.771

Table 24. MCWBNDs-II obtained through Generator 21.

v	k_1	k_2	k_3	i	Sets of Shifts	E_n
40	5	4	3	$i \pmod 4 \equiv 2$	[15,18,18,26], [7,8,9,10], [11,12,13], [5,16,17], [19,20]	0.777
88	7	6	5	$i \pmod 4 \equiv 0$	[2,3,4,7,30,41], [10,11,12,13,14,20], [17,19,21,28,36,40], [23,24,25,26,27,29], [16,31,33,34,56], [18,35,37,38,39], [42,42,43,44]	0.848

- $k_1 \pmod 4 \equiv 3$, $i \pmod 4 \equiv 0$.

Example 22. *MCWBNDs-II obtained through Generator 21. See Table 24.*

Generator 22. *MCWBNDs-II for $v = 2ik_1 + 4k_2 + 4k_3 - 2$ with $k_2 = k_1 - 1$, $k_3 = k_1 - 2$ can be obtained if:*

- $k_1 \pmod 4 \equiv 1$, $i \pmod 4 \equiv 3$.
- $k_1 \pmod 4 \equiv 3$, $i \pmod 4 \equiv 1$.

Example 23. *MCWBNDs-II obtained through Generator 22. See Table 25.*

Table 25. MCWBNDs-II obtained through Generator 22.

v	k_1	k_2	k_3	i	Sets of Shifts	E_n
56	5	4	3	$i \pmod 4 \equiv 0$	[19,25,26,36], [7,8,11,26], [12,13,14,15], [16,17,18], [9,21,23], [22,24], [27,28]	0.770
56	7	6	5	$i \pmod 4 \equiv 1$	[2,3,4,5,15,26], [11,13,22,36,21], [16,17,18,23,24], [8,10,12,19], [25,26,27,28]	0.839

6.2 For $k_2 = k_1 - 1$ and $k_3 = k_1 - 3$

Generator 23. *MCWBNDs-II for $v = 2ik_1 + 2k_2 + 2k_3 - 2$ with $k_2 = k_1 - 1$, $k_3 = k_1 - 3$ can be obtained, if:*

- $k_1 \pmod 4 \equiv 1$, $i \pmod 4 \equiv 3$.
- $k_1 \pmod 4 \equiv 3$, $i \pmod 4 \equiv 1$.

Example 24. *MCWBNDs-II obtained through Generator 23. See Table 26.*

Generator 24. *MCWBNDs-II for $v = 2ik_1 + 2k_2 + 4k_3 - 2$ with $k_2 = k_1 - 1$, $k_3 = k_1 - 3$ can be obtained if:*

Table 26. MCWBNDs-II obtained through Generator 23.

v	k_1	k_2	k_3	i	Sets of Shifts	E_n
80	9	8	6	$i \pmod 4 \equiv 3$	[4,5,6,7,22,35,38,40], [10,15,16,17,21,23,24,25], [12,13,14,18,19,20,26,27], [28,30,31,32,33,51,34], [8,36,37,38,39]	0.887
32	7	6	4	$i \pmod 4 \equiv 1$	[5,6,7,12,14,16], [8,9,10,21,13],[2,14,15]	0.845

- $k_1 = 4l$, i integer, $l > 1$.
- $k_1 = 4l + 2$, i odd.
- k_1 (odd) > 5, $i \pmod 4 \equiv 1$.

Example 25. *MCWBNDs-II obtained through Generator 24. See Table 27.*

Table 27. MCWBNDs-II obtained through Generator 24.

v	k_1	k_2	k_3	i	Sets of Shifts	E_n
48	8	7	5	integer	[2,3,4,5,6,7,20], [11,12,13,14,15,22], [16,19,22,31], [18,21,23,24]	0.856
32	6	5	3	odd	[2,4,5,6,14], [9,10,16,21], [12,13], [14,15]	0.784
40	7	6	4	$i \pmod 4 \equiv 1$	[2,3,4,5,7,18], [11,12,13,20,15], [18,19,26], [8,10,16]	0.827

Generator 25. *MCWBNDs-II for $v = 2ik_1 + 4k_2 + 2k_3 - 2$ with $k_2 = k_1 - 1$, $k_3 = k_1 - 3$ can be obtained if:*

- $k_1 = 4l + 2$, i even.
- k_1 (odd) > 5, $i \pmod 4 \equiv 3$.

Example 26. *MCWBNDs-II obtained through Generator 25. See Table 28.*

Table 28. MCWBNDs-II obtained through Generator 25.

v	k_1	k_2	k_3	i	Sets of Shifts	E_n
48	6	5	3	even	[8,9,20,22,31], [5,7,10,11,12], [4,13,14,15], [18,19,21,22], [23,24]	0.813
72	7	6	4	$i \pmod 4 \equiv 3$	[2,3,5,7,20,34], [9,10,15,23,33,46], [14,16,17,19,30,36], [21,24,25,27,34], [18,28,29,31,32], [11,22,35]	0.848

Generator 26. *MCWBNDs-II for $v = 2ik_1 + 4k_2 + 4k_3 - 2$ with $k_2 = k_1 - 1$, $k_3 = k_1 - 3$ can be obtained if:*

- $k_1 \pmod 4 \equiv 1$, $i \pmod 4 \equiv 1$.
- $k_1 \pmod 4 \equiv 3$, $i \pmod 4 \equiv 3$.

Example 27. *MCWBNDs-II obtained through Generator 26. See Table 29.*

Table 29. MCWBNDs-II obtained through Generator 26.

v	k_1	k_2	k_3	i	Sets of Shifts	E_n
72	9	8	6	$i \pmod 4 \equiv 1$	[2,4,5,6,7,8,9,30], [14,15,16,17,20,24,25], [11,12,18,19,22,23,29], [21,27,28,31,34], [33,34,35,36,46]	0.875
80	7	6	4	$i \pmod 4 \equiv 3$	[3,4,6,7,19,40], [9,10,11,12,13,17],[16,18,20,21,32,38], [23,25,26,27,37], [24,28,30,31,33], [36,39,51], [5,35,38]	0.837

6.3 For $k_2 = k_1 - 2$ and $k_3 = k_1 - 3$

Generator 27. *MCWBNDs-II for* $v = 2ik_1 + 2k_2 + 2k_3 - 2$ *with* $k_2 = k_1 - 2$, $k_3 = k_1 - 3$ *can be obtained if:*

- $k_1 = 4l + 2$, i odd & l integer.
- k_1 (odd) > 5, $i \pmod 4 \equiv 1$.

Example 28. *MCWBNDs-II obtained through Generator 27. See Table 30.*

Table 30. MCWBNDs-II obtained through Generator 27.

v	k_1	k_2	k_3	i	Sets of Shifts	E_n
24	6	4	3	odd	[4,6,9,10,16], [5,7,10], [11,12]	0.789
72	7	5	4	$i \pmod 4 \equiv 1$	[6,7,10,34,36,46], [8,9,11,12,13,15], [29,19,20,21,22,17], [14,23,24,27,25,28], [30,31,32,33], [2,34,35]	0.847

Generator 28. *MCWBNDs-II for* $v = 2ik_1 + 2k_2 + 4k_3 - 2$ *with* $k_2 = k_1 - 2$, $k_3 = k_1 - 3$ *can be obtained if:*

- $k_1 \pmod 4 \equiv 1$, $i \pmod 4 \equiv 2$.
- $k_1 \pmod 4 \equiv 3$, $i \pmod 4 \equiv 0$.

Example 29. *MCWBNDs-II obtained through Generator 28. See Table 31.*

Table 31. MCWBNDs-II obtained through Generator 28.

v	k_1	k_2	k_3	i	Sets of Shifts	E_n
72	9	8	6	$i \pmod 4 \equiv 2$	[2,3,4,6,7,8,9,32], [14,15,16,17,25,34,36,46], [11,12,21,22,23,24,31],[20,27,28,29,30], [18,19,33,34,35]	0.827
54	7	6	4	$i \pmod 4 \equiv 2$	[10,11,18,20,23,24], [8,9,14,21,25,26], [7,15,16,26,41], [12,17,19], [4,22,27]	0.833

Generator 29. *MCWBNDs-II for* $v = 2ik_1 + 4k_2 + 2k_3 - 2$ *with* $k_2 = k_1 - 2$, $k_3 = k_1 - 3$ *can be obtained if:*

- $k_1 = 4l$, i integer & $l > 1$.
- $k_1 = 4l + 2$, i odd.
- k_1 (odd) > 5, $i \pmod 4 \equiv 1$.

Example 30. *MCWBNDs-II obtained through Generator 29. See Table 32.*

Table 32. MCWBNDs-II obtained through Generator 29.

v	k_1	k_2	k_3	i	Sets of Shifts	E_n
48	8	7	5	integer	[2,3,4,5,6,8,19], [10,12,13,16,21,24], [7,11,14,15,18,31], [20,22,22,23]	0.737
32	6	5	3	odd	[2,4,5,8,12], [6,7,9,10], [13,14,16,21], [14,15]	0.632
40	7	6	4	$i \pmod 4 \equiv 0$	[4,7,11,17,18,20], [1,5,9,10,15], [8,12,16,18,26], [6,13,19]	0.692

Generator 30. *MCWBNDs-II for $v = 2ik_1 + 4k_2 + 4k_3 - 2$ with $k_2 = k_1 - 2$, $k_3 = k_1 - 3$ can be obtained if*

- $k_1 \pmod 4 \equiv 3$, $i \pmod 4 \equiv 1$.

Example 31. *MCWBNDs-II obtained through Generator 30. See Table 33.*

Table 33. MCWBNDs-II obtained through Generator 30.

v	k_1	k_2	k_3	i	Sets of Shifts	E_n
48	7	6	4	$i \pmod 4 \equiv 1$	[6,9,11,20,22,24], [5,7,10,12,14], [13,15,16,21,31], [8,18,19], [2,22,23]	0.698

6.4 For $k_2 = k_1 - 1$ and $k_3 = 3, 4, 5$

Generator 31. *MCWBNDs-II for $v = 2ik_1 + 2k_2 + 2k_3 - 2$ with $k_2 = k_1 - 1$, $k_3 = 3$ can be obtained if:*

- $k_1 \pmod 4 \equiv 1$, $i \pmod 4 \equiv 2$.
- $k_1 \pmod 4 \equiv 3$, $i \pmod 4 \equiv 0$.

Example 32. *MCWBNDs-II obtained through Generator 31. See Table 34.*

Table 34. MCWBNDs-II obtained through Generator 31.

v	k_1	k_2	k_3	i	Sets of Shifts	E_n
56	9	8	3	$i \pmod 4 \equiv 2$	[4,5,6,7,9,16,26,36], [10,11,12,13,14,15,17,18], [19,21,22,23,24,25,26], [27,28]	0.876
72	7	6	3	$i \pmod 4 \equiv 2$	[4,5,6,7,13,34], [8,10,11,12,14,15], [17,18,19,20,21,33], [25,27,28,32,34,46], [22,23,29,30,31], [35,36]	0.846

Generator 32. *MCWBNDs-II for $v = 2ik_1 + 2k_2 + 2k_3 - 2$ with $k_2 = k_1 - 1$, $k_3 = 4$ can be obtained if:*

- k_1 (odd) > 5, $i \pmod 4 \equiv 1$.
- $k_1 = 4l + 2$, i even.

Example 33. *MCWBNDs-II obtained through Generator 32. See Table 35.*

Table 35. MCWBNDs-II obtained through Generator 32.

v	k_1	k_2	k_3	i	Sets of Shifts	E_n
40	9	8	4	$i \pmod 4 \equiv 1$	[4,5,6,7,8,9,18,20], [11,12,13,16,15,17,26], [2,18,19]	0.877
40	6	5	4	even	[7,12,17,18,20], [4,5,8,9,11], [13,15,16,26], [2,18,19]	0.826

Generator 33. *MCWBNDs-II for* $v = 2ik_1 + 2k_2 + 2k_3 - 2$ *with* $k_2 = k_1 - 1$, $k_3 = 5$ *can be obtained if:*

- $k_1 \pmod 4 \equiv 1$, $i \pmod 4 \equiv 0$.
- $k_1 \pmod 4 \equiv 3$, $i \pmod 4 \equiv 2$.

Example 34. *MCWBNDs-II obtained through Generator 33. See Table 36.*

Table 36. MCWBNDs-II obtained through Generator 33.

v	k_1	k_2	k_3	i	Sets of Shifts	E_n
60	9	8	5	$i \pmod 4 \equiv 0$	[6,8,16,19,21,27,29,53], [5,11,13,14,15,17,18,24], [4,9,10,12,25,28,30], [22,23,26,29]	0.768
48	7	6	5	$i \pmod 4 \equiv 0$	[2,3,5,7,8,22], [10,11,12,13,15,31], [14,16,19,18,20], [21,22,23,24]	0.854

6.5 For $k_2 = k_1 - 2$ and $k_3 = 3, 4, 5$

Generator 34. *MCWBNDs-II for* $v = 2ik_1 + 2k_2 + 2k_3 - 2$ *with* $k_2 = k_1 - 2$, $k_3 = 3$ *can be obtained if:*

- $k_1 = 4l$, i integer & $l > 1$.
- $k_1 = 4l + 2$, i odd.
- k_1 (odd) > 5, $i \pmod 4 \equiv 3$.

Example 35. *MCWBNDs-II obtained through Generator 34. See Table 37.*

Table 37. MCWBNDs-II obtained through Generator 34.

v	k_1	k_2	k_3	i	Sets of Shifts	E_n
32	8	6	3	integer	[3,4,5,7,8,14,21], [9,10,12,13,14], [15,16]	0.846
40	10	8	3	odd	[10,5,6,7,8,9,12,18,3], [11,13,15,16,17,18,26], [19,20]	0.875
56	7	5	3	$i \pmod 4 \equiv 3$	[3,4,5,6,7,15], [9,10,11,13,12,21], [16,17,18,19], [19,22]	0.826

Generator 35. *MCWBNDs-II for* $v = 2ik_1 + 2k_2 + 2k_3 - 2$ *with* $k_2 = k_1 - 2$, $k_3 = 4$ *can be obtained if:*

- $k_1 \pmod 4 \equiv 1$, $i \pmod 4 \equiv 2$.
- $k_1 \pmod 4 \equiv 3$, $i \pmod 4 \equiv 0$.

Example 36. *MCWBNDs-II obtained through Generator 35. See Table 38.*

Table 38. MCWBNDs-II obtained through Generator 35.

v	k_1	k_2	k_3	i	Sets of Shifts	E_n
56	9	7	4	$i \pmod 4 \equiv 2$	[5,6,7,8,9,19,26,28], [10,11,12,13,14,15,16,18], [21,22,23,24,25,36], [2,26,27]	
72	7	5	4	$i \pmod 4 \equiv 0$	[5,7,12,34,36,46], [8,9,10,11,14,17], [16,20,21,22,23,29], [15,19,24,25,27,28],[30,31,32,33],[2,34,35]	0.848

Generator 36. *MCWBNDs-II for* $v = 2ik_1 + 2k_2 + 2k_3 - 2$ *with* $k_2 = k_1 - 2$, $k_3 = 5$ *can be obtained if*

- k_1 (odd) > 7, $i \pmod 4 \equiv 1$.

Example 37. *MCWBNDs-II obtained through Generator 36. See Table 39.*

Table 39. MCWBNDs-II obtained through Generator 36.

v	k_1	k_2	k_3	i	Sets of Shifts	E_n
40	9	7	5	$i \pmod 4 \equiv 0$	[4,5,6,7,8,9,18,20], [10,11,12,13,16,17], [15,18,19,26]	0.875

7 Conclusion

Some generators have been successfully developed to generate cyclic shifts for MCWBNDs-II when $m \pmod 4 \equiv 0$ & 3, in blocks of three different sizes. Efficiency of neighbor effects for each design is also evaluated which shows that these designs are efficient and economical to control neighbor effects for $v = 2ik_1 + 2k_2 + 2k_3 - 2$, $v = 2ik_1 + 2k_2 + 4k_3 - 2$, $v = 2ik_1 + 4k_2 + k_3 - 2$ and $v = 2ik_1 + 4k_2 + 4k_3 - 2$.

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Author Contributions

Jamshaid ul Hassan: Conceptualization, Methodology. **Sajid Hussain**: Data curation, Writing- Original draft preparation, Software, Supervision. **Abid Khan**: Visualization, Investigation. **Hurria Ali**: Investigation, Writing- Reviewing. **Aqsa Safdar**: Software, Validation. **Abdul Salam**: Writing- Reviewing and Editing.

Compliance with Ethical Standards

It is declare that all authors don't have any conflict of interest. It is also declare that this article does not contain any studies with human participants or animals performed by any of the authors. Furthermore, informed consent was obtained from all individual participants included in the study.

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