A GENERAL CLASS OF ESTIMATORS FOR FINITE POPULATION MEAN IN THE PRESENCE OF NON-RESPONSE WHEN USING THE SECOND RAW MOMENTS

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Revised October 2013

ABSTRACT: In this paper, a general class of family of estimators for estimation of finite population mean is proposed under non-response, by using information on second raw moments. Properties of some ratio, product, modified ratio and modified product type estimators, which are members of a suggested class of estimators, are studied. It is shown that a suggested class of estimators performs better than the usual ratio and product type estimators as well as regression and other considered estimators. A numerical study is carried out to support a suggested class of estimators.

Keywords: Non-response, Auxiliary variable, Bias, Mean square error, Second raw moments, Efficiency

1. Introduction and Symbols. In survey related to human populations, information from all units collected in a sample is in most cases not obtained even after some call backs. When the respondent and non-respondent differs from each other then the estimates obtained from the incomplete data is not only biased but also remains unknown. Hansen and Hurwitz [4] have developed a simple technique of sub-sampling, the non-respondent by more persuasions in order to adjust for the non-response in a mail survey. Generally the auxiliary information can be used to increase the precision of the estimators. When population mean of the auxiliary variable is known, in the presence of non-response, the problem of estimation of population mean of the study variable \( y \) has been dealt by Cochran [1], Rao [9], [10] and Khare and Srivastava [5], [6]. Some researchers have also used known population parameters of the auxiliary variable for improving efficiency of the estimators. For example, Sisodia and Dwivedi [13] and Pandey and Dubey [8] have used the coefficient of variation along with population mean of the auxiliary variable. Upadhya and Singh [18] and Singh et al. [14] have used the coefficient of kurtosis of the auxiliary variable in estimating the population mean of the study variable. Rao and Mudholkar [11] and Singh and Espejo [12] have introduced the ratio-cum-product type estimators for estimating the population mean. Singh and Tailor [15], [16] have used the known correlation coefficient for the estimation of population mean. Tailor and Sharma [17] introduced a modified ratio-cum-product estimator using known coefficient of variation and coefficient of kurtosis for estimation of population mean. Dubey and Uprety [3] have used second raw moments and showed that the estimator is better than the regression estimator. Consider a finite population \( \Omega = \{1, 2, \ldots, N\} \). Let \( y \) and \( x \) be the study variable and the auxiliary variable respectively taking values \( y_i \) and \( x_i \) on the \( i \)th unit of the population. Assuming that a simple random sample of size \( n \) is drawn from the population \( \Omega \) of which
only \( n_1 \) units respond and \( n_2 \) do not respond. We further assume that a population consist of two strata, those who respond at first attempt belongs to the first stratum of size \( N_1 \) and those who do not respond belongs to the second stratum of size \( N_2 \). The sample sizes \( n_1 \) and \( n_2 \) are assumed to be drawn from Stratum I and Stratum II respectively. From the \( n_2 \) non-respondents, a sub-sample of \( r = n_2 \) units are selected by simple random sample with out replacement (SRSWOR), where \( k > 1 \) is the inverse sampling rate at the second phase sample of size \( n \). Assume that all the \( r \) units will respond this time around.

Hansen and Hurwitz [4] proposed an unbiased estimator for population mean \( \bar{Y} \), given by

\[
\bar{y} = w_1 \bar{y}_1 + w_2 \bar{y}_2, \quad \text{where} \quad \bar{y}_1 = \frac{\sum_{i=1}^{n_1} y_i}{n_1}, \quad \bar{y}_2 = \frac{\sum_{i=1}^{r} y_i}{r} \quad \text{and} \quad w_i = \frac{n_i}{n} \quad \text{for} \quad i = 1, 2.
\]

The variance of \( \bar{y} \) is given by

\[
V(\bar{y}) = \left(1 - \frac{1}{n}\right) S^2_y + \frac{W_2 (k-1)}{n} S^2_{y(2)},
\]

where \( S^2_y = \frac{\sum_{i=1}^{N} (y_i - \bar{Y})^2}{N-1} \), \( S^2_{y(2)} = \frac{\sum_{i=1}^{N_2} (y_i - \bar{Y}_2)^2}{N_2-1} \), \( \bar{Y} = \frac{\sum_{i=1}^{N} y_i}{N} \) and \( \bar{Y}_2 = \frac{\sum_{i=1}^{N_2} y_i}{N_2} \).

Let \( x \) be the auxiliary variable taking values \( x_i \) on the \( i \)th unit of the population having mean \( \bar{X} = \frac{\sum_{i=1}^{N} x_i}{N} \). Let \( U \) be the second raw moments taking values \( u_i = x_i^2 \) on \( i \)th unit of population having mean \( \bar{U} = \frac{\sum_{i=1}^{N} u_i}{N} \). Let means and second raw moments of the auxiliary variable of responding and non-responding groups of population be denoted by

\[
\bar{X}_1 = \frac{\sum_{i=1}^{N_1} x_i}{N_1}, \quad \bar{X}_2 = \frac{\sum_{i=1}^{N_2} x_i}{N_2}, \quad \bar{U}_1 = \frac{\sum_{i=1}^{N_1} u_i}{N_1} \quad \text{and} \quad \bar{U}_2 = \frac{\sum_{i=1}^{N_2} u_i}{N_2} \quad \text{respectively}.
\]

Similarly an unbiased estimator for population mean and population second raw moments are given by

\[
\bar{x} = w_1 \bar{x}_1 + w_2 \bar{x}_2, \quad \text{and} \quad \bar{u} = w_1 \bar{u}_1 + w_2 \bar{u}_2,
\]

where \( \bar{x}_1 = \frac{\sum_{i=1}^{n_1} x_i}{n_1}, \quad \bar{x}_2 = \frac{\sum_{i=1}^{r} x_i}{r}, \quad \bar{u}_1 = \frac{\sum_{i=1}^{n_1} u_i}{n_1} \quad \text{and} \quad \bar{u}_2 = \frac{\sum_{i=1}^{r} u_i}{r} \) are usual sample means and sample mean of second raw moments of responding and non-responding groups based \( n_1 \) units from the first stratum and \( r \) sub-sample units from the second stratum.

The variances of \( \bar{x} \) and \( \bar{u} \) are given by

\[
V(\bar{x}) = \left(1 - \frac{1}{n}\right) S^2_x + \frac{W_2 (k-1)}{n} S^2_{x(2)} \quad \text{and} \quad V(\bar{u}) = \left(1 - \frac{1}{n}\right) S^2_u + \frac{W_2 (k-1)}{n} S^2_{u(2)},
\]
\[
S^2_x = \frac{\sum_{i=1}^N (x_i - \bar{X})^2}{N - 1}, \quad S^2_{x(2)} = \frac{\sum_{i=1}^{N_2} (x_i - \bar{X}_2)^2}{N_2 - 1}, \quad S^2_u = \frac{\sum_{i=1}^N (u_i - \bar{U})^2}{N - 1}
\]
and
\[
S^2_{u(2)} = \frac{\sum_{i=1}^{N_2} (u_i - \bar{U}_2)^2}{N_2 - 1}.
\]

To obtain the properties of estimators, we define the following error terms.

Let \( \bar{y}^* = (1 + e_2^*) \bar{Y} \), \( \bar{x}^* = (1 + e_2^*) \bar{X} \), \( \bar{x} = (1 + e_1) \bar{X} \), \( \bar{u} = (1 + e_2) \bar{U} \), such that

\[
E(e_i^*) = 0 \quad \text{for} \quad i = 0, 1, 2, \quad E(e_i^*) = 0 \quad \text{for} \quad i = 1, 2,
\]

\[
E(e_0^2) = \lambda C^2_x + \lambda^* C^2_{x(2)} = V^*_{200}, \quad E(e_1^2) = \lambda C^2_x + \lambda^* C^2_{x(2)} = V^*_{002}, \quad E(e_2^2) = \lambda C^2_x = V^*_{002},
\]

where

\[
C^2_x = \frac{S^2_x}{\bar{X}}, \quad C^2_{x(2)} = \frac{S^2_{x(2)}}{\bar{X}}, \quad C_x = \frac{S_x}{\bar{X}}, \quad C_{x(2)} = \frac{S_{x(2)}}{\bar{X}}, \quad u = \frac{S_u}{\bar{U}}, \quad C_{u(2)} = \frac{S_{u(2)}}{\bar{U}},
\]

To estimate \( \bar{y} \), we assume that \( \bar{X} \) and \( \bar{U} \) are known.

Tailor and Sharma [17] have suggested an estimator which makes use of coefficient of variation \( C_x \) and coefficient of kurtosis \( (\beta_{2(x)}) \) of \( x \), is given by

\[
\hat{Y}_S = \bar{y}^* \left[ \lambda_1 \left( \frac{C_x \bar{X} + \beta_{2(x)}}{C_x \bar{X} + \beta_{2(x)}} \right) + (1 - \lambda_1) \left( \frac{C_{x(2)} \bar{X} + \beta_{2(x)}}{C_{x(2)} \bar{X} + \beta_{2(x)}} \right) \right], (1)
\]

where \( \lambda_1 \) is the constant.

The bias and \( MSE \) of \( \hat{Y}_S \) at \( \lambda_{1 \text{opt}} = \frac{1}{2} + \frac{V^*_1}{2 \tau_2 V^*_0} \) are given by

\[
B(\hat{Y}_S) \approx \left( \frac{\tau_1}{4} (V^*_{200} + V^*_{110}) - \frac{V^*_{110}^2}{V^*_{020}} \right),
\]

where

\[
S^2_{x(2)} = \frac{\sum_{i=1}^{N_2} (x_i - \bar{X}_2)^2}{N_2 - 1}, \quad S^2_u = \frac{\sum_{i=1}^N (u_i - \bar{U})^2}{N - 1}
\]

and

\[
S^2_{u(2)} = \frac{\sum_{i=1}^{N_2} (u_i - \bar{U}_2)^2}{N_2 - 1}.
\]
where \( t_3 = \frac{C_t X}{C_t X + \beta_{2(x)}} \)
and
\[ M(\hat{Y}_3)_{min} \approx \bar{Y}^2 V_{200}^* \left( 1 - \rho^{*2} \right), \tag{3} \]
where \( \rho^* \) is the population correlation coefficient.
Similarly other authors have also suggested different ratio and product type estimators in the presence of non-response which are given in Tables 1-4.

### 2.1. Proposed class of estimators

We generalize the estimator of Tailor and Sharma [17] in which different known population parameters of the auxiliary variable \( x \) are used. For example we can use coefficient of skewness \( \phi_{3x} \), coefficient of kurtosis \( \phi_{2x} \), coefficient of correlation \( \rho_{yx} \), coefficient of variation \( C_x \) for improving the efficiency of the estimators for population mean.

Tailor and Sharma [18] generalized estimator is given by
\[ \hat{Y}_{TS} = \bar{y}^* \left[ \lambda_2 \left( \frac{A\bar{X} + B}{A\bar{X} + B} \right) + \left( 1 - \lambda_2 \right) \left( \frac{A\bar{X}^* + B}{A\bar{X} + B} \right) \right], \tag{4} \]
where \( A \) and \( B \) are some known population parameters of \( x \) and \( \lambda_2 \) is the constant whose value is to be determined.

Bias and minimum \( MSE \) of \( \hat{Y}_{TS} \), to first degree of approximation at optimum value of
\[ \lambda_2 = \frac{1}{2} + \frac{V_{110}^*}{2t_4 V_{020}^*}, \]
are given by
\[ B(\hat{Y}_{TS}) \approx \left( \frac{t_4}{4} \left( V_{020}^* + V_{110}^* \right) \right) - \frac{V_{110}^*}{V_{020}^*}, \tag{5} \]
where \( t_4 = \frac{A\bar{X}}{C_{3\bar{X} + B}} \),
and
\[ M(\hat{Y}_{TS})_{min} \approx \bar{Y}^2 V_{200}^* \left( 1 - \rho^{*2} \right), \tag{6} \]
Using the same amount of information of \( x \), we define the following estimator, where second raw moments is used, given by
\[ \hat{Y}_{M1} = \bar{y}^* \left[ \lambda_3 \left( \frac{C\bar{U}^* + D}{C\bar{U} + D} \right) + \left( 1 - \lambda_3 \right) \left( \frac{C\bar{U} + D}{C\bar{U}^* + D} \right) \right], \tag{7} \]
where \( \lambda_3 \) is the constant whose value is to be determined.

Substituting \( \hat{Y}_{TS} \) given in (4) instead of \( \bar{y}^* \) given in (7), a modified estimator becomes
\[ \hat{Y}_{M2} = \bar{y}^* \left[ \lambda_3 \left( \frac{A\bar{X} + B}{A\bar{X}^* + B} \right) + \left( 1 - \lambda_2 \right) \left( \frac{A\bar{X}^* + B}{A\bar{X} + B} \right) \right] \left[ \lambda_3 \left( \frac{C\bar{U}^* + D}{C\bar{U} + D} \right) + \left( 1 - \lambda_3 \right) \left( \frac{C\bar{U} + D}{C\bar{U}^* + D} \right) \right], \tag{8} \]
where \( A, B, C \) and \( D \) are known population parameters, \( \lambda_2 \) and \( \lambda_3 \) are constants whose values are to be determined so that the \( MSE \) of \( \hat{Y}_{M2} \) is minimum.
Now expressing (8) in terms of \( e^* \)'s, we have
\[
\hat{Y}_{M_2} = \left(1 + e_0^*\right) \bar{Y} \left[ \lambda_2 \left(1 + t_1 e_1^*\right)^{-1} + (1 - \lambda_2) \left(1 + t_4 e_4^*\right) \right] \left[ \lambda_3 \left(1 + t_5 e_5^*\right)^{-1} + (1 - \lambda_3) \left(1 + t_5 e_5^*\right) \right]
\] (9)
where \( t_5 = \frac{CU}{CU + D} \).

Expanding and ignoring powers of \( e^* \)'s greater than two in (9), we have
\[
\hat{Y}_{M_2} - \bar{Y} \cong \bar{Y} \left[ e_0^* + (1 - 2\lambda_2) t_1 e_1^* + (1 - 2\lambda_3) t_4 e_4^* + (1 - 2\lambda_2) t_2 e_2^* + (1 - 2\lambda_3) t_5 e_5^* + (1 - 2\lambda_3) t_4 e_4^* \right] \]
\[
+ \left( \lambda_2 t_1 + \lambda_3 t_4 \right) t_5 e_2^* e_2^* + \left( 1/2 \right) \lambda_2 t_1^2 + \left( 1/2 \right) \lambda_3 t_4^2 + \left( 1/2 \right) \lambda_2 t_1^2 e_2^* + \left( 1/2 \right) \lambda_3 t_4^2 e_2^* \right] .
\] (10)

Using (10), the bias and \( MSE \) of \( \hat{Y}_{M_2} \), to first degree of approximation, are given by
\[
B(\hat{Y}_{M_2}) \cong \bar{Y} \left[ (1 - 2\lambda_2) t_1 V_{110}^* + (1 - 2\lambda_3) t_4 V_{101}^* + (1 - 2\lambda_2) (1 - 2\lambda_3) t_4 t_5 V_{011}^* \right]
\]
\[
+ \left( \lambda_2 t_1 + \lambda_3 t_4 \right) t_5 V_{200}^* + \left( 1/2 \right) \lambda_2 t_1^2 V_{020}^* + \left( 1/2 \right) \lambda_3 t_4^2 V_{020}^* + \left( 1/2 \right) \lambda_2 t_1^2 V_{020}^* + \left( 1/2 \right) \lambda_3 t_4^2 V_{020}^* \right] .
\] (11)

From (12), the optimum values of \( \lambda_2 \) and \( \lambda_3 \), are given by
\[
\hat{\lambda}_2 = \frac{1}{2} + \frac{V_{020}^* V_{020}^* - V_{011}^* V_{011}^*}{t_5 (V_{020}^* V_{020}^* - V_{020}^*)} \quad \text{and} \quad \hat{\lambda}_3 = \frac{1}{2} + \frac{V_{011}^* V_{110}^* - V_{110}^* V_{110}^*}{t_4 (V_{020}^* V_{020}^* - V_{020}^*)} .
\]

Substituting the optimum values of \( \lambda_2 \) and \( \lambda_3 \) in (12), we get the minimum \( MSE \) of \( \hat{Y}_{M_2} \), given by
\[
M(\hat{Y}_{M_2})_{min} \cong \bar{Y}^2 \left[ V_{200}^* - \frac{V_{020}^* V_{020}^* + V_{020}^* V_{020}^* - 2 V_{020}^* V_{020}^*}{V_{020}^* V_{020}^* - V_{020}^*} \right].
\] (13)

After making the following substitutions,
\[
\rho^* = \phi_{110}^* = \frac{V_{110}^*}{V_{200}^* V_{020}^*}, \quad \phi_{101}^* = \frac{V_{101}^*}{(V_{200}^* V_{020}^*)^2}, \quad \phi_{011}^* = \frac{V_{011}^*}{V_{200}^*}, \quad \phi_{002}^* = \frac{V_{020}^*}{V_{020}^* V_{020}^*}
\]
\[
\psi^* = \frac{(\phi_{101}^* - \rho^* \phi_{011}^*)^2}{(\phi_{002}^* - \phi_{011}^*)^2},
\]
we have
\[
M(\hat{Y}_{M_2})_{min} \cong \bar{Y}^2 V_{200}^* \left(1 - \rho^* - \psi^*\right).
\] (14)

2.2. Various situations of non-response
We discuss the following two situations.

**Situation-I**
In Situation-I, we assume that population mean \( \bar{X} \) is known and we have incomplete information on both the study variable \( (y) \) and the auxiliary variable \( (x) \). Using (8), some modified estimators are given in Table 1.
Table 1: Some ratio and product type estimators of a suggested family of estimators given in (8) under Situation-I for $C = 0$ and $D = 1$.

<table>
<thead>
<tr>
<th>Ratio type estimator</th>
<th>Product type estimator</th>
<th>$A$</th>
<th>$B$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\lambda_2 = \lambda_3 = 1$</td>
<td>$\lambda_2 = \lambda_3 = 0$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\hat{y}_{R1} = \bar{y}^* \left( \frac{X}{\bar{x}} \right)$</td>
<td>$\hat{y}_{P1} = \bar{y}^* \left( \frac{X}{\bar{x}} \right)$</td>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td>$\hat{y}_{R2} = \bar{y}^* \left( \frac{X + C_x}{\bar{x} + C_x} \right)$</td>
<td>$\hat{y}_{P2} = \bar{y}^* \left( \frac{X + C_x}{\bar{x} + C_x} \right)$</td>
<td>1</td>
<td>$C_x$</td>
</tr>
<tr>
<td>$\hat{y}<em>{R3} = \bar{y}^* \left( \frac{\beta</em>{2(x)} X}{\beta_{2(x)} \bar{x} + C_x} \right)$</td>
<td>$\hat{y}<em>{P3} = \bar{y}^* \left( \frac{\beta</em>{2(x)} X}{\beta_{2(x)} \bar{x} + C_x} \right)$</td>
<td>$\beta_{2(x)}$</td>
<td>$C_x$</td>
</tr>
<tr>
<td>$\hat{y}<em>{R4} = \bar{y}^* \left( \frac{C_x X + \beta</em>{2(x)}}{C_x \bar{x} + \beta_{2(x)}} \right)$</td>
<td>$\hat{y}<em>{P4} = \bar{y}^* \left( \frac{C_x \bar{x} + \beta</em>{2(x)}}{C_x \bar{x} + \beta_{2(x)}} \right)$</td>
<td>$C_x$</td>
<td>$\beta_{2(x)}$</td>
</tr>
<tr>
<td>$\hat{y}_{R5} = \bar{y}^* \left( \frac{\bar{x} + S_x}{\bar{x} + S_x} \right)$</td>
<td>$\hat{y}_{P5} = \bar{y}^* \left( \frac{\bar{x} + S_x}{\bar{x} + S_x} \right)$</td>
<td>1</td>
<td>$S_x$</td>
</tr>
<tr>
<td>$\hat{y}<em>{R6} = \bar{y}^* \left( \frac{\beta</em>{1(x)} X + S_x}{\beta_{1(x)} \bar{x} + S_x} \right)$</td>
<td>$\hat{y}<em>{P6} = \bar{y}^* \left( \frac{\beta</em>{1(x)} X + S_x}{\beta_{1(x)} \bar{x} + S_x} \right)$</td>
<td>$\beta_{1(x)}$</td>
<td>$S_x$</td>
</tr>
<tr>
<td>$\hat{y}<em>{R7} = \bar{y}^* \left( \frac{\beta</em>{2(x)} X + S_x}{\beta_{2(x)} \bar{x} + S_x} \right)$</td>
<td>$\hat{y}<em>{P7} = \bar{y}^* \left( \frac{\beta</em>{2(x)} X + S_x}{\beta_{2(x)} \bar{x} + S_x} \right)$</td>
<td>$\beta_{2(x)}$</td>
<td>$S_x$</td>
</tr>
<tr>
<td>$\hat{y}<em>{R8} = \bar{y}^* \left( \frac{\bar{x} + \rho</em>{yx}}{\bar{x} + \rho_{yx}} \right)$</td>
<td>$\hat{y}<em>{P8} = \bar{y}^* \left( \frac{\bar{x} + \rho</em>{yx}}{\bar{x} + \rho_{yx}} \right)$</td>
<td>1</td>
<td>$\rho_{yx}$</td>
</tr>
<tr>
<td>$\hat{y}<em>{R9} = \bar{y}^* \left( \frac{\bar{x} + \beta</em>{2(x)}}{\bar{x} + \beta_{2(x)}} \right)$</td>
<td>$\hat{y}<em>{P9} = \bar{y}^* \left( \frac{\bar{x} + \beta</em>{2(x)}}{\bar{x} + \beta_{2(x)}} \right)$</td>
<td>1</td>
<td>$\beta_{2(x)}$</td>
</tr>
</tbody>
</table>

Biases and $MSE$s of the ratio type estimators, $\hat{y}_{Ri}^*$, $i = 1, 2, ..., 9$, to first degree of approximation, are given by

\[ B(\hat{y}_{Ri}^*) \approx \bar{y} \left[ V_i^2 \frac{V_{100}}{2} - V_i V_{110} \right] \]  

(15)

and

\[ M(\hat{y}_{Ri}^*) \approx \bar{y}^2 \left[ V_{200}^* + V_i^2 V_{100}^* - 2V_i V_{110}^* \right]. \]  

(16)

where

\[ V_1 = 1, \quad V_2 = \left( \frac{X}{X + C_x} \right), \quad V_3 = \left( \frac{\beta_{2(x)} X}{\beta_{2(x)} X + C_x} \right), \quad V_4 = \left( \frac{C_x X}{C_x X + \beta_{2(x)} } \right), \quad V_5 = \left( \frac{X}{X + S_x} \right), \]

\[ V_6 = \left( \frac{\beta_{1(x)} X}{\beta_{1(x)} X + S_x} \right), \quad V_7 = \left( \frac{\beta_{2(x)} X}{\beta_{2(x)} X + S_x} \right), \quad V_8 = \left( \frac{\bar{x}}{\bar{x} + \rho_{yx}} \right), \quad \text{and} \]

\[ V_9 = \left( \frac{\beta_{2(x)} X}{\bar{x} + \beta_{2(x)} X} \right) \]
\[ V_9 = \left( \frac{\bar{X}}{\bar{X} + \beta_{2(x)}} \right). \]

Similarly biases and MSEs of the product type estimators, \( \bar{Y}_9 \), \( i = 1, 2, \ldots, 9 \), given in Table 1, to first degree of approximation, are given by
\[ B(\bar{Y}_9) \approx \bar{Y}_9 \left[ V_{110} \right] \]  \hspace{1cm} (17)

and
\[ M(\bar{Y}_9) \approx \bar{Y}_9^2 \left[ V_{200}^* + V_{200}^* + 2V_{1100}^* \right]. \]  \hspace{1cm} (18)

**Situation-II**

In Situation-II, we assume that population mean \( \bar{X} \) is known and we have incomplete information on the study variable \( y \) but complete information on the auxiliary variable \( x \). Using (8), some modified estimators are given in Table 2

<table>
<thead>
<tr>
<th>Ratio type estimator</th>
<th>Product type estimator</th>
<th>( A )</th>
<th>( B )</th>
</tr>
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<tbody>
<tr>
<td>( \hat{Y}_{R1} ) = ( \bar{y} \left( \frac{\bar{X}}{\bar{X}} \right) )</td>
<td>( \hat{Y}_{P1} ) = ( \bar{y} \left( \frac{x}{\bar{X}} \right) )</td>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td>( \hat{Y}_{R2} ) = ( \bar{y} \left( \frac{\bar{X} + C_x}{\bar{X} + C_x} \right) )</td>
<td>( \hat{Y}_{P2} ) = ( \bar{y} \left( \frac{x + C_x}{\bar{X} + C_x} \right) )</td>
<td>1</td>
<td>( C_x )</td>
</tr>
<tr>
<td>( \hat{Y}<em>{R3} ) = ( \bar{y} \left( \frac{\beta</em>{2(x)} \bar{X} + C_x}{\beta_{2(x)} \bar{X} + C_x} \right) )</td>
<td>( \hat{Y}<em>{P3} ) = ( \bar{y} \left( \frac{\beta</em>{2(x)} \bar{X} + C_x}{\beta_{2(x)} \bar{X} + C_x} \right) )</td>
<td>( \beta_{2(x)} )</td>
<td>( C_x )</td>
</tr>
<tr>
<td>( \hat{Y}<em>{R4} ) = ( \bar{y} \left( \frac{C_x \bar{X} + \beta</em>{2(x)}}{C_x \bar{X} + \beta_{2(x)}} \right) )</td>
<td>( \hat{Y}<em>{P4} ) = ( \bar{y} \left( \frac{C_x \bar{X} + \beta</em>{2(x)}}{C_x \bar{X} + \beta_{2(x)}} \right) )</td>
<td>( C_x )</td>
<td>( \beta_{2(x)} )</td>
</tr>
<tr>
<td>( \hat{Y}_{R5} ) = ( \bar{y} \left( \frac{\bar{X} + S_x}{\bar{X} + S_x} \right) )</td>
<td>( \hat{Y}_{P5} ) = ( \bar{y} \left( \frac{\bar{X} + S_x}{\bar{X} + S_x} \right) )</td>
<td>1</td>
<td>( S_x )</td>
</tr>
<tr>
<td>( \hat{Y}<em>{R6} ) = ( \bar{y} \left( \frac{\beta</em>{1(x)} \bar{X} + S_x}{\beta_{1(x)} \bar{X} + S_x} \right) )</td>
<td>( \hat{Y}<em>{P6} ) = ( \bar{y} \left( \frac{\beta</em>{1(x)} \bar{X} + S_x}{\beta_{1(x)} \bar{X} + S_x} \right) )</td>
<td>( \beta_{1(x)} )</td>
<td>( S_x )</td>
</tr>
<tr>
<td>( \hat{Y}<em>{R7} ) = ( \bar{y} \left( \frac{\beta</em>{2(x)} \bar{X} + S_x}{\beta_{2(x)} \bar{X} + S_x} \right) )</td>
<td>( \hat{Y}<em>{P7} ) = ( \bar{y} \left( \frac{\beta</em>{2(x)} \bar{X} + S_x}{\beta_{2(x)} \bar{X} + S_x} \right) )</td>
<td>( \beta_{2(x)} )</td>
<td>( S_x )</td>
</tr>
<tr>
<td>( \hat{Y}<em>{R8} ) = ( \bar{y} \left( \frac{\bar{X} + \rho</em>{yx}}{\bar{X} + \rho_{yx}} \right) )</td>
<td>( \hat{Y}<em>{P8} ) = ( \bar{y} \left( \frac{\bar{X} + \rho</em>{yx}}{\bar{X} + \rho_{yx}} \right) )</td>
<td>1</td>
<td>( \rho_{yx} )</td>
</tr>
<tr>
<td>( \hat{Y}<em>{R9} ) = ( \bar{y} \left( \frac{\bar{X} + \beta</em>{2(x)}}{\bar{X} + \beta_{2(x)}} \right) )</td>
<td>( \hat{Y}<em>{P9} ) = ( \bar{y} \left( \frac{\bar{X} + \beta</em>{2(x)}}{\bar{X} + \beta_{2(x)}} \right) )</td>
<td>1</td>
<td>( \beta_{2(x)} )</td>
</tr>
</tbody>
</table>
The expressions for biases and MSEs of the estimators $\hat{Y}_{Ri}^*$ and $\hat{Y}_{pi}^*$, $i=1, 2, \ldots, 9$, given in Table 2 under Situation-II will be according to (15)-(18) with the following slight substitutions:

$V_{020}$ for $V_{020}^*$, $V_{002}$ for $V_{002}^*$, $V_{110}$ for $V_{110}^*$, $V_{011}$ for $V_{011}^*$, $V_{101}$ for $V_{101}^*$ and $V_{200}$ for $V_{200}^*$.

### 2.3. Comparison of estimators

**Situation-I**

(i) The suggested estimator $\hat{Y}_{M_i}$ will perform better than ratio type estimators $\hat{Y}_{Ri}$ $(i = 1, 2, \ldots, 9)$, if

$$M(\hat{Y}_{Ri}) - M(\hat{Y}_{M_i})_{\min} > 0 \Rightarrow \left(V_{1} \sqrt{V_{020}^*} - \rho \sqrt{V_{200}^*}\right)^2 + V_{200}^* \psi^2 > 0.$$  \hspace{1cm} (19)

(ii) The suggested estimator $\hat{Y}_{M_i}$ will perform better than product type estimators $\hat{Y}_{pi}$ $(i = 1, 2, \ldots, 9)$, if

$$M(\hat{Y}_{pi}) - M(\hat{Y}_{M_i})_{\min} > 0 \Rightarrow \left(V_{1} \sqrt{V_{020}^*} + \rho \sqrt{V_{200}^*}\right)^2 + V_{200}^* \psi^2 > 0.$$  \hspace{1cm} (20)

(iii) The suggested estimator $\hat{Y}_{M_i}$ will perform better than estimator $\hat{Y}_{TS}$ if

$$M(\hat{Y}_{TS}) - M(\hat{Y}_{M_i})_{\min} > 0 \Rightarrow \psi^2 > 0.$$  \hspace{1cm} (21)

**Situation-II**

(i) The suggested estimator $\hat{Y}_{M_i}$ will perform better than ratio type estimators $\hat{Y}_{Ri}$ $(i = 1, 2, \ldots, 9)$, if

$$M(\hat{Y}_{Ri}) - M(\hat{Y}_{M_i})_{\min} > 0 \Rightarrow \left(V_{1} \sqrt{V_{020}^*} - \rho \sqrt{V_{200}^*}\right)^2 + V_{200}^* \psi^2 > 0.$$  \hspace{1cm} (22)

(ii) The suggested estimator $\hat{Y}_{M_i}$ will perform better than product type estimators $\hat{Y}_{pi}$ $(i = 1, 2, \ldots, 9)$, if

$$M(\hat{Y}_{pi}) - M(\hat{Y}_{M_i})_{\min} > 0 \Rightarrow \left(V_{1} \sqrt{V_{020}^*} + \rho \sqrt{V_{200}^*}\right)^2 + V_{200}^* \psi^2 > 0.$$  \hspace{1cm} (23)

(iii) The suggested estimator $\hat{Y}_{M_i}$ will perform better than $\hat{Y}_{TS}$ if

$$M(\hat{Y}_{TS}) - M(\hat{Y}_{M_i})_{\min} > 0 \Rightarrow \psi^2 > 0.$$  \hspace{1cm} (24)

Conditions (19)-(24) are obviously true.

### 2.4. Some other proposed estimators

The estimator $\hat{Y}_{M_i}$ in (8) reduces to the following estimators by substituting $\lambda_2 = 1$ and $\lambda_2 = 0$ as

$$\hat{Y}_{RM} = \hat{Y}^* \left[ \left( \frac{AX + B}{AX^* + B} \right) \left[ \lambda_3 \left( \frac{CU^* + D}{CU + D} \right) + (1 + \lambda_3) \left( \frac{CU + D}{CU^* + D} \right) \right] \right], \text{ for } \lambda_2 = 1,$$  \hspace{1cm} (25)

and
\[ \hat{Y}_{PM} = \bar{Y} \left[ \left( \frac{A\hat{X} + B}{A\hat{X} + B} \right) \lambda_3 \left( \frac{C\hat{U} + D}{C\hat{U} + D} \right) + \left( + \lambda_3 \right) \left( \frac{C\hat{U} + D}{C\hat{U} + D} \right) \right], \text{for } \lambda_2 = 0. \]  

(26)

The bias and \textit{MSE} of \( \hat{Y}_{RM} \), to the first degree of approximation at optimum value of \( \lambda_3 \) i.e. \( \lambda_{3(\text{opt})} = \frac{1}{2} + \frac{V_{101}^* - t_3 V_{011}^*}{2t_4 V_{002}^*} \), are given by

\[ B(\hat{Y}_{RM}) \approx \left[ -t_4 V_{110}^* + (1 - 2\lambda_3) t_3 V_{101}^* - (1 - 2\lambda_3) t_4 V_{011}^* + \frac{t_4^2}{2} V_{020}^* + \frac{\lambda_3 t_4^2}{2} V_{002}^* \right] \]  

(27)

and

\[ M(\hat{Y}_{RM})_{\text{min}} = V^2 \left[ V_{200}^* + t_4^2 V_{020}^* - 2t_4 V_{110}^* - (V_{101}^* - t_4 V_{011}^*)^2 \right]. \]  

(28)

Similarly bias and \textit{MSE} of \( \hat{Y}_{PM} \), to the first degree of approximation at optimum value of \( \lambda_3 \) i.e. \( \lambda_{3(\text{opt})} = \frac{1}{2} + \frac{V_{101}^* + t_3 V_{011}^*}{2t_4 V_{002}^*} \), are given by

\[ B(\hat{Y}_{PM}) \approx \left[ t_4 V_{110}^* + (1 - 2\lambda_3) t_3 V_{101}^* + (1 - 2\lambda_3) t_4 V_{011}^* + \frac{\lambda_3 t_4^2}{2} V_{020}^* \right] \]  

(29)

and

\[ M(\hat{Y}_{PM})_{\text{min}} = V^2 \left[ V_{200}^* + t_4^2 V_{020}^* + 2t_4 V_{110}^* - (V_{101}^* + t_4 V_{011}^*)^2 \right]. \]  

(30)

Under Situation-II the estimator \( \hat{Y}_{M_2} \), in (8) reduces to the following estimators by substituting \( \lambda_2 = 1 \) and \( \lambda_2 = 0 \) as

\[ \hat{Y}_{RM} = \bar{Y} \left[ \left( \frac{A\hat{X} + B}{A\hat{X} + B} \right) \lambda_3 \left( \frac{C\hat{U} + D}{C\hat{U} + D} \right) + \left( + \lambda_3 \right) \left( \frac{C\hat{U} + D}{C\hat{U} + D} \right) \right], \text{for } \lambda_2 = 1 \]  

(31)

and

\[ \hat{Y}_{PM} = \bar{Y} \left[ \left( \frac{A\hat{X} + B}{A\hat{X} + B} \right) \lambda_3 \left( \frac{C\hat{U} + D}{C\hat{U} + D} \right) + \left( + \lambda_3 \right) \left( \frac{C\hat{U} + D}{C\hat{U} + D} \right) \right] \text{ for } \lambda_2 = 0. \]  

(32)

The functional form of the expressions for biases and \textit{MSEs} will remain as given in (27)-(30) with the following slight substitutions:

- \( V_{020} \) for \( V_1^* \), \( V_{002} \) for \( V_0^* \), \( V_{110} \) for \( V_{10}^* \), \( V_{011} \) for \( V_{01}^* \), \( V_{101} \) for \( V_{10}^* \) and \( V_{002} \) for \( V_{20}^* \).

Using (8), some ratio and product type estimators of a suggested class of estimators under Situations I and II are given in Tables 3-6.
Table 3: Using (8) some ratio type members of a suggested family of estimators under Situation-I for $\lambda_2 = 1$.

<table>
<thead>
<tr>
<th>Ratio type estimator</th>
<th>$A$</th>
<th>$B$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\hat{Y}_{RM1}^* = \bar{y}^* \left( \frac{\bar{x}}{\bar{X}} \right) \left[ \lambda_3 \left( \frac{C\bar{u}^* + D}{C\bar{u}^* + D} \right) + (1 - \lambda_3) \left( \frac{C\bar{U} + D}{C\bar{u}^* + D} \right) \right]$</td>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td>$\hat{Y}_{RM2}^* = \bar{y}^* \left( \frac{\bar{x} + C_x}{\bar{x} + C_x} \right) \left[ \lambda_3 \left( \frac{C\bar{u}^* + D}{C\bar{u}^* + D} \right) + (1 - \lambda_3) \left( \frac{C\bar{U} + D}{C\bar{u}^* + D} \right) \right]$</td>
<td>1</td>
<td>$C_x$</td>
</tr>
<tr>
<td>$\hat{Y}<em>{RM3}^* = \bar{y}^* \left( \frac{\beta</em>{2(x)} \bar{x} + C_x}{\beta_{2(x)} \bar{x} + C_x} \right) \left[ \lambda_3 \left( \frac{C\bar{u}^* + D}{C\bar{u}^* + D} \right) + (1 - \lambda_3) \left( \frac{C\bar{U} + D}{C\bar{u}^* + D} \right) \right]$</td>
<td>$\beta_{2(x)}$</td>
<td>$C_x$</td>
</tr>
<tr>
<td>$\hat{Y}<em>{RM4}^* = \bar{y}^* \left( \frac{C_x \bar{x} + \beta</em>{2(x)}}{C_x \bar{x} + \beta_{2(x)}} \right) \left[ \lambda_3 \left( \frac{C\bar{u}^* + D}{C\bar{u}^* + D} \right) + (1 - \lambda_3) \left( \frac{C\bar{U} + D}{C\bar{u}^* + D} \right) \right]$</td>
<td>$C_x$</td>
<td>$\beta_{2(x)}$</td>
</tr>
<tr>
<td>$\hat{Y}_{RM5}^* = \bar{y}^* \left( \frac{\bar{x} + S_x}{\bar{x} + S_x} \right) \left[ \lambda_3 \left( \frac{C\bar{u}^* + D}{C\bar{u}^* + D} \right) + (1 - \lambda_3) \left( \frac{C\bar{U} + D}{C\bar{u}^* + D} \right) \right]$</td>
<td>1</td>
<td>$S_x$</td>
</tr>
<tr>
<td>$\hat{Y}<em>{RM6}^* = \bar{y}^* \left( \frac{\beta</em>{1(x)} \bar{x} + S_x}{\beta_{1(x)} \bar{x} + S_x} \right) \left[ \lambda_3 \left( \frac{C\bar{u}^* + D}{C\bar{u}^* + D} \right) + (1 - \lambda_3) \left( \frac{C\bar{U} + D}{C\bar{u}^* + D} \right) \right]$</td>
<td>$\beta_{1(x)}$</td>
<td>$S_x$</td>
</tr>
<tr>
<td>$\hat{Y}<em>{RM7}^* = \bar{y}^* \left( \frac{\beta</em>{2(x)} \bar{x} + S_x}{\beta_{2(x)} \bar{x} + S_x} \right) \left[ \lambda_3 \left( \frac{C\bar{u}^* + D}{C\bar{u}^* + D} \right) + (1 - \lambda_3) \left( \frac{C\bar{U} + D}{C\bar{u}^* + D} \right) \right]$</td>
<td>$\beta_{2(x)}$</td>
<td>$S_x$</td>
</tr>
<tr>
<td>$\hat{Y}<em>{RM8}^* = \bar{y}^* \left( \frac{\bar{x} + \rho</em>{3x}}{\bar{x} + \rho_{3x}} \right) \left[ \lambda_3 \left( \frac{C\bar{u}^* + D}{C\bar{u}^* + D} \right) + (1 - \lambda_3) \left( \frac{C\bar{U} + D}{C\bar{u}^* + D} \right) \right]$</td>
<td>1</td>
<td>$\rho_{3x}$</td>
</tr>
<tr>
<td>$\hat{Y}<em>{RM9}^* = \bar{y}^* \left( \frac{\bar{x} + \beta</em>{2(x)}}{\bar{x} + \beta_{2(x)}} \right) \left[ \lambda_3 \left( \frac{C\bar{u}^* + D}{C\bar{u}^* + D} \right) + (1 - \lambda_3) \left( \frac{C\bar{U} + D}{C\bar{u}^* + D} \right) \right]$</td>
<td>1</td>
<td>$\beta_{2(x)}$</td>
</tr>
</tbody>
</table>

Table 4: Using (8) some product type members of a suggested family of estimators under Situation-I for $\lambda_2 = 0$.

<table>
<thead>
<tr>
<th>Product type estimator</th>
<th>$A$</th>
<th>$B$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\hat{Y}_{PM1}^* = \bar{y}^* \left( \frac{\bar{x}^<em>}{\bar{x}} \right) \left[ \lambda_3 \left( \frac{C\bar{u}^</em> + D}{C\bar{u}^* + D} \right) + (1 - \lambda_3) \left( \frac{C\bar{U} + D}{C\bar{u}^* + D} \right) \right]$</td>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td>$\hat{Y}_{PM2}^* = \bar{y}^* \left( \frac{\bar{x}^* + C_x}{\bar{x} + C_x} \right) \left[ \lambda_3 \left( \frac{C\bar{u}^* + D}{C\bar{u}^* + D} \right) + (1 - \lambda_3) \left( \frac{C\bar{U} + D}{C\bar{u}^* + D} \right) \right]$</td>
<td>1</td>
<td>$C_x$</td>
</tr>
<tr>
<td>$\hat{Y}<em>{PM3}^* = \bar{y}^* \left( \frac{\beta</em>{2(x)} \bar{x}^* + C_x}{\beta_{2(x)} \bar{x} + C_x} \right) \left[ \lambda_3 \left( \frac{C\bar{u}^* + D}{C\bar{u}^* + D} \right) + (1 - \lambda_3) \left( \frac{C\bar{U} + D}{C\bar{u}^* + D} \right) \right]$</td>
<td>$\beta_{2(x)}$</td>
<td>$C_x$</td>
</tr>
</tbody>
</table>
Biases and MSEs of the estimators \( \hat{Y}_{pmi} \) and \( \hat{Y}_{rmi} \), \( i = 1, 2, \ldots \), given in Tables 3 and 4, to first degree of approximation, are given by

\[
B(\hat{Y}_{pmi}) \approx \bar{Y} \left[ V_i^2 \frac{V_{020}^*}{2} - V_i V_{110}^* + (1 - 2\lambda_3) V_i V_{101}^* - (1 - 2\lambda_3) V_i V_{011}^* + \frac{\lambda_3 t_5^2}{2} V_{002}^* \right],
\]

\[
M(\hat{Y}_{pmi}) = \bar{Y}^2 \left[ V_{200}^* + V_i^2 V_{020}^* - 2V_i V_{110}^* - (V_{101}^* - V_{011}^*)^2 / V_{002}^* \right],
\]

\[
B(\hat{Y}_{rmi}) = \bar{Y} \left[ V_i V_{110}^* + (1 - 2\lambda_3) V_i V_{101}^* + (1 - 2\lambda_3) V_i V_{011}^* + \frac{\lambda_3 t_5^2}{2} V_{002}^* \right],
\]

and

\[
M(\hat{Y}_{rmi}) = \bar{Y}^2 \left[ V_{200}^* + V_i^2 V_{020}^* + 2V_i V_{110}^* - (V_{101}^* + V_{011}^*)^2 / V_{002}^* \right].
\]

Table 5: Using (8) some ratio type estimators of a suggested family of estimators under Situation-II for \( \lambda_2 = 1 \).

<table>
<thead>
<tr>
<th>Ratio type estimator</th>
<th>A</th>
<th>B</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \hat{Y}_{rmi1} )</td>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td>( \hat{Y}_{rmi2} )</td>
<td>( C_x )</td>
<td>( \beta_{2(x)} )</td>
</tr>
<tr>
<td>( \hat{Y}_{rmi3} )</td>
<td>( \beta_{2(x)} )</td>
<td>( C_x )</td>
</tr>
</tbody>
</table>
\[ \hat{Y}_{RM4} = \bar{y} \left( \frac{C_X\bar{X} + \beta_{2(x)}}{C_X\bar{X} + \beta_{2(x)}} \right) \lambda_3 \left( \frac{C_U + D}{C_U + D} \right) + (1 - \lambda_3) \left( \frac{C_U + D}{C_U + D} \right) \] 

\[ \hat{Y}_{RM5} = \bar{y} \left( \frac{\bar{X} + S_x}{\bar{X} + S_x} \right) \lambda_3 \left( \frac{C_U + D}{C_U + D} \right) + (1 - \lambda_3) \left( \frac{C_U + D}{C_U + D} \right) \] 

\[ \hat{Y}_{RM6} = \bar{y} \left( \frac{\beta_{1(x)}\bar{X} + S_x}{\beta_{1(x)}\bar{X} + S_x} \right) \lambda_3 \left( \frac{C_U + D}{C_U + D} \right) + (1 - \lambda_3) \left( \frac{C_U + D}{C_U + D} \right) \] 

\[ \hat{Y}_{RM7} = \bar{y} \left( \frac{\beta_{2(x)}\bar{X} + S_x}{\beta_{2(x)}\bar{X} + S_x} \right) \lambda_3 \left( \frac{C_U + D}{C_U + D} \right) + (1 - \lambda_3) \left( \frac{C_U + D}{C_U + D} \right) \] 

\[ \hat{Y}_{RM8} = \bar{y} \left( \frac{\bar{X} + \rho_{yx}}{\bar{X} + \rho_{yx}} \right) \lambda_3 \left( \frac{C_U + D}{C_U + D} \right) + (1 - \lambda_3) \left( \frac{C_U + D}{C_U + D} \right) \] 

\[ \hat{Y}_{RM9} = \bar{y} \left( \frac{\bar{X} + \beta_{2(x)}}{\bar{X} + \beta_{2(x)}} \right) \lambda_3 \left( \frac{C_U + D}{C_U + D} \right) + (1 - \lambda_3) \left( \frac{C_U + D}{C_U + D} \right) \]
The expressions for biases and MSE of the estimators \( \hat{Y}_{RMi} \) and \( \hat{Y}_{PMi} \) \( i=1, 2, \ldots \) under Situation-II will remain as given in (33)-(36) with the following slight substitutions

\[
V_{020} \text{ for } V_{020}^{*}, \ V_{002} \text{ for } V_{002}^{*}, \ V_{110} \text{ for } V_{110}^{*}, \ V_{011} \text{ for } V_{011}^{*}, \ V_{101} \text{ for } V_{101}^{*} \text{ and } V_{200} \text{ for } V_{200}^{*}.
\]

2.6. Comparison of estimators

Under Situation-I, the estimators \( \hat{Y}_{RMi}^{*} \) \( (i = 1, 2, \ldots , 9) \) given in Table 3 will perform better than usual ratio type estimators \( \hat{Y}_{Ri}^{*} \) \( (i = 1, 2, \ldots , 9) \) given in Table 1 if

\[
M(\hat{Y}_{Ri}^{*}) - M(\hat{Y}_{RMi}^{*}) > 0 \Rightarrow \frac{(V_{101}^{*} - V_{101}^{*})}{V_{002}^{*}} > 0. \tag{37}
\]

The estimators \( \hat{Y}_{PMi}^{*} \) \( (i = 1, 2, \ldots , 9) \) given in Table 4 will perform better than usual product type estimators \( \hat{Y}_{Pi}^{*} \) \( \xi = 1, 2, \ldots 9 \) given in Table 1 if

\[
M(\hat{Y}_{Pi}^{*}) - M(\hat{Y}_{PMi}^{*}) > 0 \Rightarrow \frac{(V_{101}^{*} + V_{101}^{*})}{V_{002}^{*}} > 0. \tag{38}
\]

The expressions for comparison of proposed estimators \( \hat{Y}_{RM}^{*} \) and \( \hat{Y}_{PM}^{*} \) given in (25) and (26) with usual ratio type estimators \( \hat{Y}_{Ri}^{*} \) \( (i = 1, 2, \ldots , 9) \) given in Table 1 and usual product type estimators \( \hat{Y}_{Pi}^{*} \) \( \xi = 1, 2, \ldots 9 \) given in Table 1, will be same as given in (37) and (38) with slight substitution of \( t_{4} \) for \( V_{i} \) \( (i = 1, 2, \ldots , 9) \).

The expressions for efficiency comparison under Situation-II will remain same as mentioned in (35)-(36) with the following substitutions

\[
V_{020} \text{ for } V_{020}^{*}, \ V_{002} \text{ for } V_{002}^{*}, \ V_{110} \text{ for } V_{110}^{*}, \ V_{011} \text{ for } V_{011}^{*}, \ V_{101} \text{ for } V_{101}^{*} \text{ and } V_{200} \text{ for } V_{200}^{*}.
\]

2.7. Numerical example

The data sets are given in Tables 7 and 8.

Population-I

Source: [Das and Tripathi [2]]

Let \( y \) be the number of agricultural laborers and \( x \) be the population of villages. The first 16 units of the population are assumed non-respondents.

<table>
<thead>
<tr>
<th>( \bar{N} ) = 96</th>
<th>( \bar{N}_{2} = 16 )</th>
<th>( n = 30 ), ( n_{2} = 12 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \bar{Y} = 137.93 )</td>
<td>( \bar{X} = 181.76 )</td>
<td>( \bar{U} = 3717295.6 )</td>
</tr>
<tr>
<td>( S_{y}^{2} = 33306.7 )</td>
<td>( S_{x}^{2} = 3684258.9 )</td>
<td>( S_{u}^{2} = 45894736841 )</td>
</tr>
<tr>
<td>( S_{y(2)}^{2} = 25092.3 )</td>
<td>( S_{x(2)}^{2} = 5524847.3 )</td>
<td>( S_{u(2)}^{2} = 52571278705.6 )</td>
</tr>
<tr>
<td>( S_{xy} = 316407.9 )</td>
<td>( S_{yu} = 35101147.4 )</td>
<td>( S_{xu} = 370269789.5 )</td>
</tr>
<tr>
<td>( S_{y(2)} = 355841.9 )</td>
<td>( S_{y(2)} = 33447821.2 )</td>
<td>( S_{x(2)} = 504390133.3 )</td>
</tr>
</tbody>
</table>
\[ \rho_{yx} = 0.903247 \quad \rho_{yu} = 0.897789 \quad \rho_{ux} = 0.900454 \]
\[ \rho_{yx(2)} = 0.955711 \quad \rho_{yu(2)} = 0.920923 \quad \rho_{ux(2)} = 0.935906 \]
\[ V_1 = 1 \quad V_2 = 0.945090 \quad V_3 = 0.996907 \]
\[ V_4 = 0.990337 \quad V_5 = 0.0865029 \quad V_6 = 0.241348 \]
\[ V_7 = 0.639452 \quad V_8 = 0.995056 \quad V_9 = 0.906582 \]

**Population-II**

**Source:** [Murthy [7]]

Let \( y \) be the output of the factory and \( x \) be the number of workers working in the factory. We randomly selected a sample of size 20 from population of size 80 and considered this as the stratum of non-respondents.

Table 8: Summary statistics for Population-II.

<table>
<thead>
<tr>
<th>( N = 80 )</th>
<th>( N_2 = 20 )</th>
<th>( n = 30 ), ( n_2 = 12 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \bar{Y} = 5182.6 )</td>
<td>( \bar{X} = 285.13 )</td>
<td>( \bar{U} = 153514 )</td>
</tr>
<tr>
<td>( S^2_y = 3369642 )</td>
<td>( S^2_x = 73132.1 )</td>
<td>( S^2_y = 66013595417 )</td>
</tr>
<tr>
<td>( S^2_{yx(2)} = 2800048 )</td>
<td>( S^2_{yu(2)} = 76595.88 )</td>
<td>( S^2_{ux(2)} = 68803364254 )</td>
</tr>
<tr>
<td>( S_{yx} = 454211 )</td>
<td>( S_{yu} = 380054066 )</td>
<td>( S_{ux} = 66781955 )</td>
</tr>
<tr>
<td>( S_{yx(2)} = 437594.9 )</td>
<td>( S_{yu(2)} = 381467001 )</td>
<td>( S_{ux(2)} = 70737846 )</td>
</tr>
<tr>
<td>( \rho_{yx} = 0.915 )</td>
<td>( \rho_{yu} = 0.806 )</td>
<td>( \rho_{ux} = 0.961 )</td>
</tr>
<tr>
<td>( \rho_{yx(2)} = 0.9449 )</td>
<td>( \rho_{yu(2)} = 0.8691 )</td>
<td>( \rho_{ux(2)} = 0.9974 )</td>
</tr>
<tr>
<td>( V_1 = 1 )</td>
<td>( V_2 = 0.9966846 )</td>
<td>( V_3 = 0.9990719 )</td>
</tr>
<tr>
<td>( V_4 = 0.986932 )</td>
<td>( V_5 = 0.513226 )</td>
<td>( V_6 = 0.6319197 )</td>
</tr>
<tr>
<td>( V_7 = 0.7905918 )</td>
<td>( V_8 = 0.9968012 )</td>
<td>( V_9 = 0.9875971 )</td>
</tr>
</tbody>
</table>

The results are given in Tables 9-12.

Table 9: Percentage relative efficiency of different estimators with to usual mean estimator under Situation-I.

<table>
<thead>
<tr>
<th>Estimator</th>
<th>Population-I</th>
<th>Population-II</th>
</tr>
</thead>
<tbody>
<tr>
<td>( (2,6) )</td>
<td>( (3,4) )</td>
<td>( (2,6) )</td>
</tr>
<tr>
<td>( (3,4) )</td>
<td>( (4,3) )</td>
<td>( (4,3) )</td>
</tr>
<tr>
<td>( \hat{Y}_{R1} )</td>
<td>1.696</td>
<td>1.532</td>
</tr>
<tr>
<td>( \hat{Y}_{R2} )</td>
<td>1.925</td>
<td>1.737</td>
</tr>
<tr>
<td>( \hat{Y}_{R3} )</td>
<td>1.708</td>
<td>1.542</td>
</tr>
<tr>
<td>( \hat{Y}_{R4} )</td>
<td>1.734</td>
<td>1.565</td>
</tr>
<tr>
<td>( \hat{Y}_{R5} )</td>
<td>482.092</td>
<td>522.705</td>
</tr>
<tr>
<td>( \hat{Y}_{R6} )</td>
<td>65.212</td>
<td>56.974</td>
</tr>
</tbody>
</table>
From Table 9, it is evident that the performance of a proposed estimator \( \hat{\mu}_{R} \) is better as compared to all other considered estimators. Further the efficiency of the estimator increases with an increase in value of \( k \) for the Population-I and decreases for the Population-II.

Table 10: Percentage relative efficiency of different estimators with respect to usual mean estimator under Situation-II.

<table>
<thead>
<tr>
<th>Estimator</th>
<th>Population-I</th>
<th>Population-II</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \hat{\mu}_{R1} )</td>
<td>2.343 (2,6)</td>
<td>46.986 (2,6)</td>
</tr>
<tr>
<td>( \hat{\mu}_{R2} )</td>
<td>2.660 (3,4)</td>
<td>47.331 (3,4)</td>
</tr>
<tr>
<td>( \hat{\mu}_{R3} )</td>
<td>2.359 (4,3)</td>
<td>47.082 (4,3)</td>
</tr>
<tr>
<td>( \hat{\mu}_{R4} )</td>
<td>2.394 (2,6)</td>
<td>43.808 (2,6)</td>
</tr>
<tr>
<td>( \hat{\mu}_{R5} )</td>
<td>286.99 (3,4)</td>
<td>48.365 (3,4)</td>
</tr>
<tr>
<td>( \hat{\mu}_{R6} )</td>
<td>83.677 (4,3)</td>
<td>145.18 (4,3)</td>
</tr>
<tr>
<td>( \hat{\mu}_{R7} )</td>
<td>6.566 (2,6)</td>
<td>76.802 (2,6)</td>
</tr>
<tr>
<td>( \hat{\mu}_{R8} )</td>
<td>2.369 (3,4)</td>
<td>47.319 (3,4)</td>
</tr>
<tr>
<td>( \hat{\mu}_{R9} )</td>
<td>2.922 (4,3)</td>
<td>48.294 (4,3)</td>
</tr>
<tr>
<td>( \hat{\mu}_{TS} )</td>
<td>322.449 (2,6)</td>
<td>171.24 (2,6)</td>
</tr>
<tr>
<td>( \hat{\mu}<em>{M</em>{2}} )</td>
<td>359.395 (3,4)</td>
<td>182.20 (3,4)</td>
</tr>
</tbody>
</table>

Note: Figures in (.,.) represents \((k, r)\).

From Table 10, we can see that the proposed estimator \( \hat{\mu}_{M_{2}} \) is more efficient as compared to other considered estimators for different values of \( k \) and \( r \) under Situation-II and also efficiency of estimators decreases with an increase in the value of \( k \) for both populations.
Table 11: Percentage relative efficiency of different estimators with respect to usual mean estimator under Situation-I.

<table>
<thead>
<tr>
<th>Estimator</th>
<th>Population-I</th>
<th></th>
<th></th>
<th></th>
<th>Population-II</th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>(2,6)</td>
<td>(3,4)</td>
<td>(4,3)</td>
<td></td>
<td>(2,6)</td>
<td>(3,4)</td>
<td>(4,3)</td>
<td></td>
</tr>
<tr>
<td>(\hat{Y}_{RM1}^*)</td>
<td>8.605</td>
<td>8.235</td>
<td>8.014</td>
<td></td>
<td>211.028</td>
<td>149.353</td>
<td>125.106</td>
<td></td>
</tr>
<tr>
<td>(\hat{Y}_{RM2}^*)</td>
<td>9.689</td>
<td>9.273</td>
<td>9.023</td>
<td></td>
<td>212.644</td>
<td>150.492</td>
<td>126.059</td>
<td></td>
</tr>
<tr>
<td>(\hat{Y}_{RM3}^*)</td>
<td>8.661</td>
<td>8.289</td>
<td>8.066</td>
<td></td>
<td>211.479</td>
<td>149.671</td>
<td>125.372</td>
<td></td>
</tr>
<tr>
<td>(\hat{Y}_{RM4}^*)</td>
<td>8.782</td>
<td>8.405</td>
<td>8.179</td>
<td></td>
<td>217.484</td>
<td>153.910</td>
<td>128.917</td>
<td></td>
</tr>
<tr>
<td>(\hat{Y}_{RM5}^*)</td>
<td>640.186</td>
<td>648.040</td>
<td>662.726</td>
<td></td>
<td>582.861</td>
<td>477.919</td>
<td>426.840</td>
<td></td>
</tr>
<tr>
<td>(\hat{Y}_{RM6}^*)</td>
<td>173.288</td>
<td>167.203</td>
<td>164.517</td>
<td></td>
<td>491.940</td>
<td>372.010</td>
<td>320.16</td>
<td></td>
</tr>
<tr>
<td>(\hat{Y}_{RM7}^*)</td>
<td>22.163</td>
<td>21.208</td>
<td>20.653</td>
<td></td>
<td>347.248</td>
<td>249.069</td>
<td>209.710</td>
<td></td>
</tr>
<tr>
<td>(\hat{Y}_{RM8}^*)</td>
<td>8.695</td>
<td>8.322</td>
<td>8.098</td>
<td></td>
<td>212.586</td>
<td>150.452</td>
<td>126.025</td>
<td></td>
</tr>
<tr>
<td>(\hat{Y}_{RM9}^*)</td>
<td>10.576</td>
<td>10.121</td>
<td>9.849</td>
<td></td>
<td>217.150</td>
<td>153.674</td>
<td>128.720</td>
<td></td>
</tr>
</tbody>
</table>

Note: Figures in (. , .) represents \((k, r)\).

Table 12: Percentage relative efficiency of different estimator with respect to usual mean estimator under Situation-II.

<table>
<thead>
<tr>
<th>Estimator</th>
<th>Population-I</th>
<th></th>
<th></th>
<th></th>
<th>Population-II</th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>(2,6)</td>
<td>(3,4)</td>
<td>(4,3)</td>
<td></td>
<td>(2,6)</td>
<td>(3,4)</td>
<td>(4,3)</td>
<td></td>
</tr>
<tr>
<td>(\hat{Y}_{RM1}^*)</td>
<td>10.836</td>
<td>12.303</td>
<td>13.723</td>
<td></td>
<td>277.381</td>
<td>204.355</td>
<td>173.9221</td>
<td></td>
</tr>
<tr>
<td>(\hat{Y}_{RM2}^*)</td>
<td>12.179</td>
<td>13.800</td>
<td>15.363</td>
<td></td>
<td>278.028</td>
<td>204.635</td>
<td>174.091</td>
<td></td>
</tr>
<tr>
<td>(\hat{Y}_{RM3}^*)</td>
<td>10.905</td>
<td>12.381</td>
<td>13.808</td>
<td></td>
<td>277.562</td>
<td>204.433</td>
<td>173.969</td>
<td></td>
</tr>
<tr>
<td>(\hat{Y}_{RM4}^*)</td>
<td>11.055</td>
<td>12.549</td>
<td>13.992</td>
<td></td>
<td>279.910</td>
<td>205.447</td>
<td>174.579</td>
<td></td>
</tr>
<tr>
<td>(\hat{Y}_{RM5}^*)</td>
<td>352.15</td>
<td>263.32</td>
<td>220.78</td>
<td></td>
<td>299.949</td>
<td>213.819</td>
<td>179.552</td>
<td></td>
</tr>
<tr>
<td>(\hat{Y}_{RM6}^*)</td>
<td>165.815</td>
<td>152.397</td>
<td>143.524</td>
<td></td>
<td>310.410</td>
<td>218.002</td>
<td>181.992</td>
<td></td>
</tr>
<tr>
<td>(\hat{Y}_{RM7}^*)</td>
<td>27.272</td>
<td>30.212</td>
<td>32.9231</td>
<td></td>
<td>307.583</td>
<td>216.884</td>
<td>181.342</td>
<td></td>
</tr>
<tr>
<td>(\hat{Y}_{RM8}^*)</td>
<td>10.947</td>
<td>12.428</td>
<td>13.8604</td>
<td></td>
<td>278.006</td>
<td>204.625</td>
<td>174.085</td>
<td></td>
</tr>
<tr>
<td>(\hat{Y}_{RM9}^*)</td>
<td>13.274</td>
<td>15.016</td>
<td>16.6904</td>
<td></td>
<td>279.783</td>
<td>205.392</td>
<td>174.546</td>
<td></td>
</tr>
</tbody>
</table>

Note: Figures in (. , .) represents \((k, r)\).

From Tables 11 and 12, it is observed that the efficiency of proposed estimators \(\hat{Y}_{RMi}^*\) and \(\hat{Y}_{RMi}\) \((i = 1, \ldots, 9)\) which uses second raw moments have increased significantly as compared to usual ratio type estimators \(\hat{Y}_R\) \((i = 1, \ldots, 9)\) under both Situation-I and Situation-II. From Table 11, it is observed that the efficiency of estimator \(\hat{Y}_{RM5}^*\) increases as the value of \(k\) increases while for rest of estimators, the
efficiency decreases as the value of $k$ increases in both populations under Situation-I. From Table 12, for Population-I, one can observe that the efficiency of estimators $\hat{y}_{RM5}$, $\hat{y}_{RM6}$ and $\hat{y}_{RM7}$ decreases as the value of $k$ increases, while for other estimators it increases. For Population-II, the efficiency decreases when value of $k$ increases.

3. Conclusion
The proposed estimator $\hat{y}_M$ which uses second raw moments is found more efficient than other considered estimators including regression and ratio-cum-product type estimators. Modified ratio type estimators $\hat{y}_{RMi}$ and $\hat{y}_{Ri}$ ($i = 1, \ldots, 9$) which uses second raw moments are also better than usual ratio type estimators $\hat{y}_{Ri}$ ($i = 1, \ldots, 9$). The use of second raw moments in association with population mean of auxiliary variable, improve the efficiency of estimators.

REFERENCES