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NUMERICAL SOLUTIONS OF SEVIRS MODEL BY MESHLESS AND FINITE DIFFERENCE METHODS

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Abstract. In this paper an operator splitting method is being considered for numerical solution of compartmental epidemiological population models with and without di usion. This is based on meshless and nite di erence method. A one step ex- plicit meshless procedure is also applied for the numerical solution of the model. The compartmental model comprises of susceptible, vaccinated, exposed, infected, recovered (SVEIRS) classes of the population. E ects of the di usion on the simulation results of the model will be studied. Due to non-availability of the exact and precise solution, the numerical results obtained are mutually compared and their correctness will be veri ed by the theoretical results .

Keywords: , Meshless methods, Finite difference methods, Radial basis functions, System of PDEs, Operator splitting, Epidemiological models, Influenza.

1 Introduction

Influenza, also known as flu, is a respiratory disease which is caused by a virus, named orthomyxovirus. Millions of people are suffer or lost their lives due to serious complexity of the infectious disease every year. Appropriate measures are being taken to control this epidemic disease. Elderly people and children are easy victims to this disease. There are three types of Influenza virus naming influenza A; influenza B; and influenza C. Influenza A infection, usually, reaches to the peak during the winter season and involve 10% or more of the population with the rate of 50% to 70% in school going children. Influenza B accounts for only 3% of all flu cases in the United States [12, 1, 5]. The influenza A is the most complicated, severe and dangerous for human being. The Influenza A mostly specific HA and NA subtypes frequently caused the dangerous epidemic diseases which include all pandemic diseases of the last century[11, 2].
Every year approximately 50000 people lost their lives due to influenza. In $20th$ century influenza pandemic outbroke many times- 1918, 1957 and 1968. During the period from 1918 to 1919, due to a flu pandemic, approximately one third of the world population were infected. The death rate was 2.3% to 5% in the affected area [3].
Mathematical models have become significant tools to analyze the spreading and controlling of infectious diseases. The model formulation process clarifies assumptions, variables, and parameters; moreover, model provide conceptual results such as thresholds, basic reproduction numbers, contact numbers, and replacement numbers. Mathematical models and computer simulations are vital instrument for engineering and testing theories, evaluating quantitative speculation, addressing specific questions, determining sensitivities to changes in parameter values, and estimating important parameters from data.

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Mathematical models are used in comparing, implementing, planning, assessing, prevention, optimizing various detection, and control programs. Epidemiology modeling can contribute the designing and analyzing of epidemiological surveys which suggest important data that should be collected to identify trends, to make general forecasts, and estimate the Uncertainty in forecasts [4, 6].
Various methods are used in order to engineer a clear, realistic, and feasible epidemic model for the dynamics of influenza. These models are developed with a view to observe the epidemiological patterns. Similarly, these models also predict the consequences of the introduction of public health interventions to control the spread of diseases. The realistic epidemic models lack the vaccination programs viewed by Dietz and Schenzle [10]. There are different sorts of epidemic models which can be improved with vaccination programs. Among these some models recommend same rate of vaccination belonging to different ages and some models consider also vaccination of given fractions of individuals at given ages [11, 8, 13, 14].
In this paper, our focus is on the numerical simulation of the SVEIRS model by three types of numerical methods. They include meshless explicit method (MEM), meshless operator splitting method (MOSM) and finite difference operator splitting method (the FDOSM). This helps us to analyze the disease dynamics in a better way.

2 The SVEIRS model in ODEs form

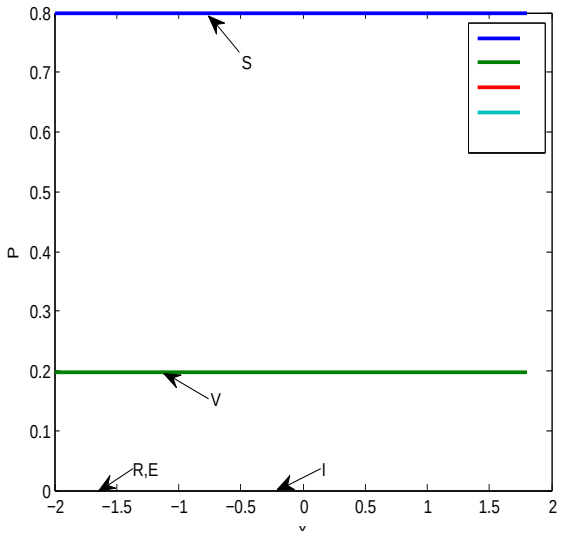
The SVEIRS model with five Classes in ODEs form is given by

$$\begin{aligned}
\frac{dS}{dt} &= -\beta\beta_E ES - \beta\beta_I IS + \alpha IS - \phi S - rS + \delta R + \theta V + r, \\
\frac{dV}{dt} &= -\beta\beta_E\beta_V EV - \beta\beta_I\beta_V IV + \alpha IS - rV - \theta V + \phi S, \\
\frac{dE}{dt} &= \beta\beta_E ES + \beta\beta_I IS - \beta\beta_E\beta_V EV - \beta\beta_I\beta_V IV - (r + \kappa + \sigma)E, \\
\frac{dI}{dt} &= \sigma E - (r + \alpha + \gamma)I + \alpha I^2, \\
\frac{dR}{dt} &= \kappa E + \gamma I - rR - \delta R + \alpha IR.
\end{aligned} \tag{1}$$

with the following condition

$$S + V + E + I + R = 1. \tag{2}$$

The values of the parameters and their biological meaning are given in the following Tab. (1) [9]:



Initial condition (6)

Figure 1: Profile of the initial conditions (6))

Table 1: **Model parameters and their biological meaning**

parameters	description	value	source
β	Contact rate	0.514000000	
β_E	Ability to cause infection by exposed individuals($0 \leqslant \beta_E \leqslant 1$)	0.250000000	
β_I	Ability to cause infection by infectious individuals($0 \leqslant \beta_I \leqslant 1$)	1.0000000000	
β_V	Tendency of vaccinated people to get infection ($0 \leqslant \beta_V \leqslant 1$)	variable.	
$1-\beta_V$	Factor by which the vaccine reduces infection($0 \leqslant \beta_V \leqslant 1$)	Variable	
σ^{-1}	Mean duration of latency(days)	2.000000000	
γ^{-1}	Mean recovery time for clinically ill(days)	5.000000000	
δ^{-1}	Duration of immunity loss(days)	365.00000000	
μ	Natural mortality rate	5.5×10^{-8}	
r	Birth rate	7.140×10^{-5}	
κ	Recovery rate of latent	1.857×10^{-4}	
α	Flu induced mortality rate	90300×10^{-6}	
θ^{-1}	Duration of vaccine induced immunity loss(days)	365.00000000	
ϕ	Rate of vaccination	Variable	

3 The SVEIRS model with diffusion

In the case when diffusion is added, the ODE model (1) is converted into a system of PDEs which is given by [9]:

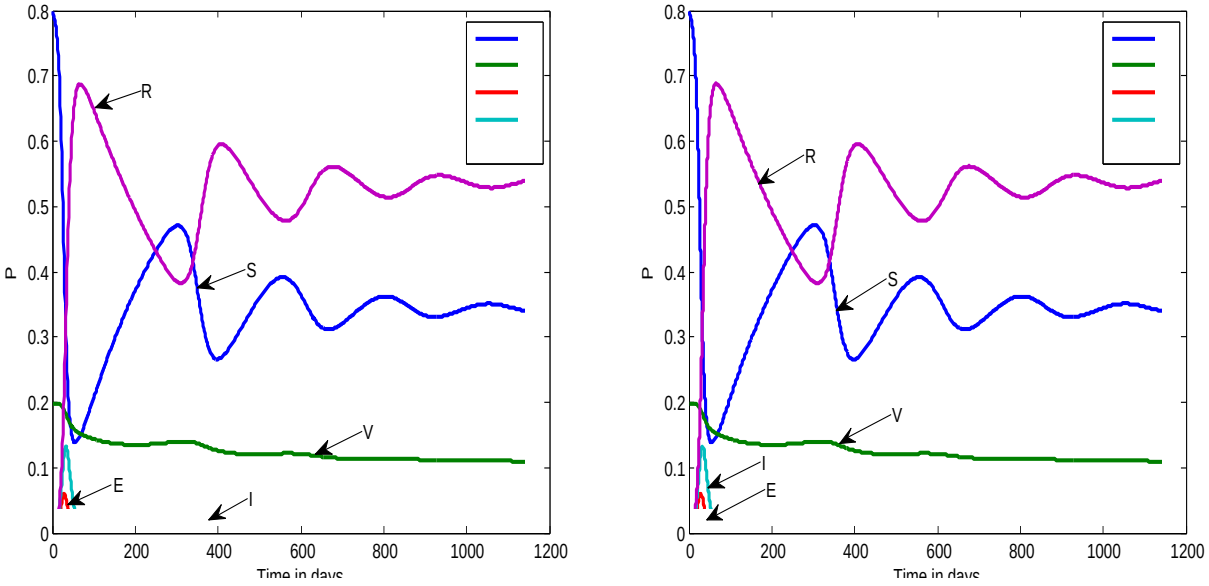
$$\begin{aligned}
\frac{dS}{dt} &= -\beta\beta_E ES - \beta\beta_I IS + \alpha IS - \phi S - rS + \delta R + \theta V + r + d_1\frac{\partial^2 S}{\partial x^2}, \\
\frac{dV}{dt} &= -\beta\beta_E\beta_V EV - \beta\beta_I\beta_V IV + \alpha IS - rV - \theta V + \phi S + d_2\frac{\partial^2 V}{\partial x^2}, \\
\frac{dE}{dt} &= \beta\beta_E ES + \beta\beta_I IS - \beta\beta_E\beta_V EV - \beta\beta_I\beta_V IV - (r+\kappa+\sigma)E + d_3\frac{\partial^2 E}{\partial x^2}, \\
\frac{dI}{dt} &= \sigma E - (r+\alpha+\gamma)I + \alpha I^2 + d_4\frac{\partial^2 I}{\partial x^2}, \\
\frac{dR}{dt} &= \kappa E + \gamma I - rR - \delta R + \alpha IR + d_5\frac{\partial^2 R}{\partial x^2}.
\end{aligned} \tag{3}$$

where d_1, d_2, d_3, d_4, and d_5 are diffusivity constants of susceptible, vaccinated, exposed, infected and recovered classes of the population respectively.
All computational domain for the above problem is taken as [-2,2]. No flux boundary conditions are used for the above system which are given as:

$$\frac{\partial S(-2,t)}{\partial x} = \frac{\partial V(-2,t)}{\partial x} = \frac{\partial E(-2,t)}{\partial x} = \frac{\partial I(-2,t)}{\partial x} = \frac{\partial R(-2,t)}{\partial x} = 0 \tag{4}$$

and

$$\frac{\partial S(2,t)}{\partial x} = \frac{\partial V(2,t)}{\partial x} = \frac{\partial E(2,t)}{\partial x} = \frac{\partial I(2,t)}{\partial x} = \frac{\partial R(2,t)}{\partial x} = 0. \tag{5}$$



Without diffusion With diffusion

Figure 2: Graphical results for the FDOSM for ICs (6), Case 1(a)

4 Initial condition

The initial condition given in [7, 9] are being used in the present numerical simulation to test the efficacy of different numerical methods. The condition is given as:

$$\begin{aligned} S_0 &= 0.799, \quad x \in [-2, 2], \\ V_0 &= 0.197, \quad x \in [-2, 2], \\ E_0 &= 0, \quad x \in [-2, 2], \\ I_0 &= 0.004, \quad x \in [-2, 2], \\ R_0 &= 0, \quad x \in [-2, 2], \end{aligned} \tag{6}$$

The graph of initial conditions (6) is shown in Fig. 1. Initial condition (6) has constant values throughout the domain which can be seen in Fig. 1.

5 Numerical Scheme

In this section, we use MOSM, MEM and the FDOSM for the numerical solution of the model model given in (3). In each case, spatial step $\triangle x = 0.1$ and time step $\triangle t = 0.024$ days are used keeping in view the Von Neumann analysis for stability. The values of diffusivity constants used in all the cases are $d_1 = 0.05$,$d_2 = 0.05$, $d_3 = 0.025$, $d_4 = 0.001$ and $d_5 = 0.0$.

6 Numerical Solutions

Initial conditions given in Eq. (6) is being considered. The parameters values as mentioned in Tab. 2 are used in numerical solution of the model given in Eqs. (1)-(3). The analysis of all cases are conducted

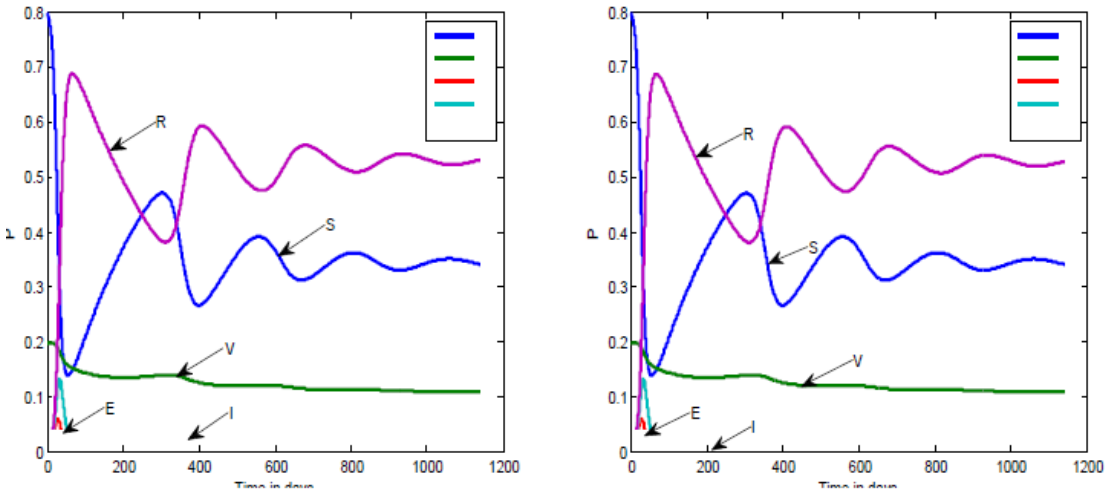
at the specific points i.e. $x = 0.0$ and $x = 1.0$ for 1200 days.

Table 2: Points of equilibrium

case	β_v	φ	points of equilibrium: disease-free equilibrium	endemic equilibrium
1(a)	0.1	0.001	(0.734846, 0.265154, 0.0, 0.0, 0.0)	(0.343293, 0.107505, 0.002986, 0.007463, 0.538753)
(b)	−−	0.002	(0.580836, 0.419164, 0.0, 0.0, 0.0)	(0.332717, 0.213269, 0.002469, 0.006169, 0.445377)
(c)	−−	0.003	(0.480195, 0.519805, 0.0, 0.0, 0.0)	(0.322322, 0.317215, 0.001960, 0.004898, 0.353605)
2(a	0.2	0.001	(0.734846, 0.265154, 0.0, 0.0, 0.0)	(0.335656, 0.091938, 0.003112, 0.007779, 0.561516)
(b)	−−	0.002	(0.580836, 0.419164, 0.0, 0.0, 0.0)	(0.318129, 0.179573, 0.002731, 0.006825, 0.492741)
(c)	−−	0.003	(0.480195, 0.519805, 0.0, 0.0, 0.0)	(0.301473, 0.262851, 0.002369, 0.005920, 0.427386)
3(a)	0.3	0.001	(0.734846, 0.265154, 0.0, 0.0, 0.0)	(0.330074, 0.079897, 0.003208, 0.008017, 0.578803)
(b)	−−	0.002	(0.580836, 0.265154, 0.0, 0.0, 0.0)	(0.307989, 0.153515, 0.002928, 0.007317, 0.528251)
(c)	−−	0.003	(0.480195, 0.519805, 0.0, 0.0, 0.0)	(0.287708, 0.221119, 0.002671, 0.006674, 0.481829)
4	1.0	0.000	(1.000000, 0.000000, 0.0, 0.0, 0.0)	(0.354043, 0.000000, 0.003512, 0.008777, 0.633667)

6.1 Initial condition (6) for the case 1(a) at $x = 0$

The simulation results of the FDOSM with the initial condition (6) for the case 1(a) with and without diffusion are shown in Fig.2 . No significant difference in the outputs of the model with and without diffusion is being observed. The numerical results of the MOSM and the MEM corresponding to same initial condition are shown in Fig.3 . As clear from these figures, the numerical results produced by the FDOSM and the MOSM resemble for each class of population. The simulated results of the class R in the case of the MOSM and MEM is little smaller in value than its counter part the FDOSM. Diffusion process has no effect on the simulation results produced by the FDOSM whereas in the case of the MOSM and the MEM, one can see mild effects of the diffusion on the classes S, V, R, E, I of the population. The overall the effects of diffusion are not prominent for each class of the population due to uniform distribution of the initial population.



With diffusion With diffusion

Figure 3: Graphical results for the MOSM (Left)and the MEM (Right) for ICs (6), Case 1(a)

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