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Upper Bound Sequences of Rotationally Symmetric Triangular Prism Constructed as Halin Graph Using Local Fractional Metric Dimension

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Abstract In this paper, we consider rotationally symmetric traingular planar network with possible planar symmetries. We find local fractional metric dimension of planar symmetries. The objective is to search sequences of local fractional metric dimension of triangular prism planar networks by joining different copies. We propose and prove generalized formulas of all sequences for local fractinal metric dimension over triangular prism..

keywordsTriangular Prism, Halin Graph, Local Fractional Metric Dimension

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1 Introduction

Graph theory has played a very important role in many branches of science. The idea of metric in graph theory, is used to find a minimum distance between different vertices in real applied problems. In a graph network, the concepts of complexity, modularity, centrality, accessibility, connectivity, robustness, and clustering are, in fact, the structural properties of that networks. Resolving sets are used to identify the source of diffusion across a network. A graph G is a network over a set of vertices with set of edges. Set of vertices is represented by $V(G)$ and set of edges is denoted by $E(G)$. While, $d(u,v)$ denotes the minimum distance between u and v. Let $M = [\{m_1, m_2, ..., m_k\}$ be a subset of vertices of a graph G. A vertex x of a graph G is said to resolve two vertices u and v of G if $d(x,u) \neq d(x,v)$. A set M is called resolving set(or locating) of G if every vertex in G can be uniquely determined by its distances from the vertices in M. Resolving set of minimum cardinality is called metric basis. The metric dimension Of G is denoted by $\beta(G)$ and is the cardinality of minimum resolving set. The minimum cardinality of local resolving set is called local metric dimension of G and it is denoted by $lmd(G)$. Resolving sets for graphs were introduced independently by Slater(1975) and Harary and Malter (1976), while the concept of resolving set and that of metric dimension were defined much earlier in a more general context of metric spaces by Blumenthal in his monography theory and applications of distance geometry [1,2]. Slater used the idea of locating sets and introduced the term resolving set for connected graphs [3,5]. Metric dimension of generalized peterson graphs has been investigated in [6]. Liuetal investigated many different aspects of graph theory such as indices, metric dimension, domination number in [7-10]. In 2000, Chartrand et al., found the solution in term of metric dimension of integer programming solution given in [11]. Further, Currie and Oellarmann introduced the concept of fractional metric dimension for getting more accurate solution of integer programming problem [12]. In [13-26], authors have already proposed several classes of integers based on partitions of integers together with graph structures and their applications in graph labeling. In this paper, we propose and prove generalized formulas of all sequences for local fractional metric dimension over triangular prism.

2 Preliminaries

In this section, we review the fundamental concepts of connected graphs, resolving function, minimal resolving function and the related terminology.

Definition 1. *Let G be a connected graph. A vertex u is said to resolve a pair of vertices if $d(x,u) \neq d(y,u)$ for every pair of vertices $x,y \in V(G)$. The resolving neighborhood of a pair of vertices $x,y \in V(G)$ is the set $\Re$={$u \in V(G) \,|\, d(x,u) \neq d(x,v)$}.*

Definition 2. *Let G be a connected graph. A function $\theta : V(G) \longrightarrow [0,1]$ is defined as resolving function of G if $\theta(\Re\{x,y\}) \geq 1$ for all $x,y \in V(G)$, where $\theta(\Re\{x,y\})=\sum_{u\in\Re\{x,y\}} \theta(u)$.*

Definition 3. *A function is a resolving function θ' of G if $\theta(u)$ is minimal resolving function of G and if $\neq \theta'(u)$ with $\theta : V(G) \longrightarrow [0,1]$ such that $\theta \leq \theta'$ and $u \in V(G)$ is not a resolving function of G.*

Definition 4. *Let θ' ba a minimal resolving function of G. Then the fractional metric dimension is defined as $dim_f(G)$ = min{$|\ \theta'\ |$: θ' is the minimal resolving function of G where $|\ \theta'\ |$ can be defined as*

$$|\ \theta'\ | = \sum_{u\in V(G)} \theta'(u).$$

Definition 5. *A resolving function is called local resolving function if $\theta'(\Re\{x,y\}) \geq 1$ and the fractional metric dimension is called local fractional metric dimension if we consider those resolving sets having adjacent pair of vertices.*

Remark 1. *The following are the well-known generalized results proved earlier.*

- *The metric dimension of a graph G is 1 if and only if G is a path.*
- *The metric dimension of an n vertex graph is $n-1$ if and only if it is a complete graph.*
- *Let G be a finite connected graph of order $n \geq 2$, then $ldim_f(G) \leq dim_f(G)$*

- *Let G be a finite connected graph of order* $n \geq 2$, *then* $\frac{n}{n-ldim(G)+1} \leq ldim_f(G) \leq \frac{n}{l(G)} \leq \frac{n}{2}$
- $ldim_f(G)$=1 *if and only if the graph G is bipartite.*
- $ldim_f(G)$=$\frac{n}{2}$ *if and only if each vertex in* $V(G)$ *has a true twin vertex.*

3 Rotationally symmetric triangular prism planar networks

In this section, we define triangular prism network with possible faces and compute local fractional metric dimension of those possible planar faces.

Proposition 1. *Let* P_1 *be a triangular prism constructed as a Halin graph from six vertex tree having five faces. Then local fractional metric dimension of* P_1 *is given by*

$$ldim_f(P_1) = \frac{3}{2}.. \tag{1}$$

Proof. Let P_1 be a triangular prism having 5 faces and let t_1^1,t_2^1,t_3^1,t_4^1,t_5^1,t_6^1 be the vertices of prism P_1. Now we are going to find the local fractional metric dimension of triangular prism P_1. So, by definition of local fractional metric dimension, all possible resolving sets are given by

$$\begin{aligned}
\bullet\, \Re\{t_1^1, t_3^1\} &= V(P_1) - \{t_5^1, t_6^1\},\\
\bullet\, \Re\{t_1^1, t_2^1\} &= V(P_1) - \phi,\\
\bullet\, \Re\{t_1^1, t_5^1\} &= V(P_1) - \{t_3^1, t_4^1\},\\
\bullet\, \Re\{t_2^1, t_4^1\} &= V(P_1) - \{t_5^1, t_6^1\},\\
\bullet\, \Re\{t_2^1, t_6^1\} &= V(P_1) - \{t_3^1, t_4^1\},\\
\bullet\, \Re\{t_3^1, t_4^1\} &= V(P_1) - \phi,\\
\bullet\, \Re\{t_3^1, t_5^1\} &= V(P_1) - \{t_1^1, t_2^1\},\\
\bullet\, \Re\{t_4^1, t_6^1\} &= V(P_1) - \{t_1^1, t_2^1\},\\
\bullet\, \Re\{t_5^1, t_6^1\} &= V(P_1) - \phi.
\end{aligned} \tag{2}$$

Since, all sets are the resolving sets but our interest is in least cardinality. Hence the minimum cardinality of resolving set is 4. So by relation, we have

$$ldim_f(P_1) = \frac{6}{4} = \frac{3}{2}. \tag{3}$$

□

Proposition 2. *let* P_2 *be a Triangular prism constructed as a Halin graph from six vertex tree having six faces. Then the local fractional metric dimension of* P_2 *is given by*

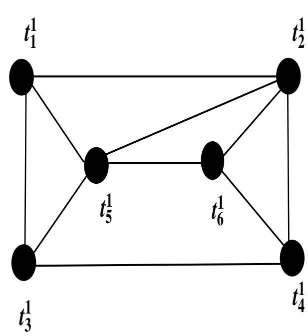
$$ldim_f(P_2) = 2. \tag{4}$$

Proof. Let P_2 be a triangular prism having 6 faces and let t_1^1,t_2^1,t_3^1,t_4^1,t_5^1,t_6^1 be the vertices of prism P_2. Now we are going to find the local fractional metric dimension of triangular prism P_2. So, by definition of local fractional metric dimension, all possible resolving sets are given by

$$\begin{aligned}
\bullet\, \Re\{t_1^1, t_3^1\} &= V(P_2) - \{t_5^1, t_6^1\},\\
\bullet\, \Re\{t_1^1, t_2^1\} &= V(P_2) - \{t_5^1\},\\
\bullet\, \Re\{t_1^1, t_5^1\} &= V(P_2) - \{t_3^1, t_2^1, t_4^1\},\\
\bullet\, \Re\{t_2^1, t_5^1\} &= V(P_2) - \{t_1^1, t_6^1\},\\
\bullet\, \Re\{t_2^1, t_4^1\} &= V(P_2) - \{t_6^1\},\\
\bullet\, \Re\{t_2^1, t_6^1\} &= V(P_2) - \{t_3^1, t_4^1, t_5^1\},\\
\bullet\, \Re\{t_3^1, t_4^1\} &= V(P_2) - \phi,\\
\bullet\, \Re\{t_3^1, t_5^1\} &= V(P_2) - \{t_1^1\},\\
\bullet\, \Re\{t_4^1, t_6^1\} &= V(P_2) - \{t_1^1, t_2^1\},\\
\bullet\, \Re\{t_5^1, t_6^1\} &= V(P_2) - \{t_2^1\}.
\end{aligned} \tag{5}$$

Since, all sets are the resolving sets but our interest is in least cardinality. Hence the minimum cardinality of resolving set is 3. So by relation, we have

$$ldim_f(P_2) = \frac{6}{3} = 2. \tag{6}$$



P_2

□

Proposition 3. *let P_3 be a Triangular prism constructed as a Halin graph from six vertex tree having six faces. Then the local fractional metric dimension of P_3 is given by*

$$ldim_f(P_3) = 2. \tag{7}$$

Proof. Let P_3 be a triangular prism having 6 faces and let t_1^1,t_2^1,t_3^1,t_4^1,t_5^1,t_6^1 be the vertices of prism P_3. Now we are going to find the local fractional metric dimension of triangular prism P_3. So, by definition of local fractional metric dimension, all possible resolving sets are given by

$$\begin{aligned}
&\bullet \Re\{t_1^1, t_3^1\} &=& \quad V(P_3) - \{t_5^1\}, \\
&\bullet \Re\{t_1^1, t_2^1\} &=& \quad V(P_3) - \{t_6^1\}, \\
&\bullet \Re\{t_1^1, t_5^1\} &=& \quad V(P_3) - \{t_3^1, t_4^1, t_6^1\}, \\
&\bullet \Re\{t_2^1, t_6^1\} &=& \quad V(P_3) - \{t_1^1, t_3^1, t_4^1\}, \\
&\bullet \Re\{t_2^1, t_4^1\} &=& \quad V(P_3) - \{t_5^1, t_6^1\}, \\
&\bullet \Re\{t_3^1, t_4^1\} &=& \quad V(P_3) - \phi, \\
&\bullet \Re\{t_3^1, t_5^1\} &=& \quad V(P_3) - \{t_1^1, t_2^1\}, \\
&\bullet \Re\{t_4^1, t_6^1\} &=& \quad V(P_3) - \{t_2^1\}, \\
&\bullet \Re\{t_5^1, t_6^1\} &=& \quad V(P_3) - \{t_1^1\}, \\
&\bullet \Re\{t_1^1, t_6^1\} &=& \quad V(P_3) - \{t_2^1, t_5^1\}.
\end{aligned} \tag{8}$$

Since, all sets are the resolving sets but our interest is in least cardinality. Hence the minimum cardinality of resolving set is 3. So by relation, we have

$$ldim_f(P_3) \quad = \quad \frac{6}{3}. \tag{9}$$

□

Proposition 4. *let P_4 be a Triangular prism constructed as a Halin graph from six vertex tree having six faces. Then the local fractional metric dimension of P_4 is given by*

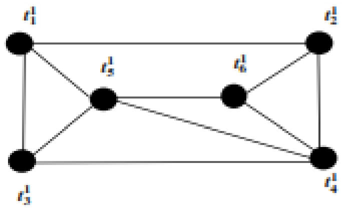
$$ldim_f(P_4) \quad = \quad 2. \tag{10}$$

Proof. Let P_4 be a triangular prism having 6 faces and let t_1^1,t_2^1,t_3^1,t_4^1,t_5^1,t_6^1 be the vertices of prism P_4. Now we are going to find the local fractional metric dimension of triangular prism P_4. So, by definition of local fractional metric dimension, all possible resolving sets are given by

$$\begin{aligned}
&\bullet \Re\{t_1^1, t_3^1\} &=& \quad V(P_4) - \{t_5^1, t_6^1\}, \\
&\bullet \Re\{t_1^1, t_2^1\} &=& \quad V(P_4) - \phi, \\
&\bullet \Re\{t_1^1, t_5^1\} &=& \quad V(P_4) - \{t_3^1\}, \\
&\bullet \Re\{t_4^1, t_5^1\} &=& \quad V(P_4) - \{t_3^1, t_6^1\}, \\
&\bullet \Re\{t_2^1, t_4^1\} &=& \quad V(P_4) - \{t_6^1\}, \\
&\bullet \Re\{t_2^1, t_6^1\} &=& \quad V(P_4) - \{t_3^1, t_4^1\}, \\
&\bullet \Re\{t_3^1, t_4^1\} &=& \quad V(P_4) - \{t_5^1\}, \\
&\bullet \Re\{t_3^1, t_5^1\} &=& \quad V(P_4) - \{t_1^1, t_2^1, t_4^1\}, \\
&\bullet \Re\{t_4^1, t_6^1\} &=& \quad V(P_4) - \{t_1^1, t_2^1, t_5^1\}, \\
&\bullet \Re\{t_5^1, t_6^1\} &=& \quad V(P_4) - \{t_4^1\}.
\end{aligned} \tag{11}$$

Since, all sets are the resolving sets but our interest is in least cardinality. Hence the minimum cardinality of resolving set is 3. So by relation, we have

$$ldim_f(P_4) \quad = \quad \frac{6}{3}. \tag{12}$$



P_4

□

Proposition 5. *let P_5 be a Triangular prism constructed as a Halin graph from six vertex tree having six faces. Then the local fractional metric dimension of P_5 is given by*

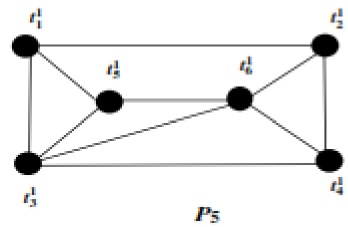
$$ldim_f(P_5) \quad = \quad 2. \tag{13}$$

Proof. Let P_5 be a triangular prism having 6 faces and let t_1^1,t_2^1,t_3^1,t_4^1,t_5^1,t_6^1 be the vertices of prism P_5. Now we are going to find the local fractional metric dimension of triangular prism P_5. So, by definition of local fractional metric dimension, all possible resolving sets are given by

$$\begin{aligned}
\bullet\, \Re\{t_1^1, t_3^1\} &= V(P_5) - \{t_5^1\}, \\
\bullet\, \Re\{t_1^1, t_2^1\} &= V(P_5) - \phi, \\
\bullet\, \Re\{t_1^1, t_5^1\} &= V(P_5) - \{t_3^1, t_4^1\}, \\
\bullet\, \Re\{t_3^1, t_6^1\} &= V(P_5) - \{t_4^1, t_5^1\}, \\
\bullet\, \Re\{t_2^1, t_4^1\} &= V(P_5) - \{t_5^1, t_6^1\}, \\
\bullet\, \Re\{t_2^1, t_6^1\} &= V(P_5) - \{t_4^1\}, \\
\bullet\, \Re\{t_3^1, t_4^1\} &= V(P_5) - \{t_6^1\}, \\
\bullet\, \Re\{t_3^1, t_5^1\} &= V(P_5) - \{t_1^1, t_2^1, t_6^1\}, \\
\bullet\, \Re\{t_4^1, t_6^1\} &= V(P_5) - \{t_1^1, t_2^1, t_3^1\}, \\
\bullet\, \Re\{t_5^1, t_6^1\} &= V(P_5) - \{t_3^1\}.
\end{aligned} \tag{14}$$

Since, all sets are the resolving sets but our interest is in least cardinality. Hence the minimum cardinality of resolving set is 3. So by relation, we have

$$ldim_f(P_5) \quad = \quad \frac{6}{3}. \tag{15}$$



□

Proposition 6. *let P_6 be a Triangular prism constructed as a Halin graph from six vertex tree having seven faces. Then the local fractional metric dimension of P_6 is given by*

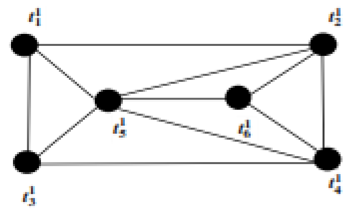
$$ldim_f(P_6) = 2. \tag{16}$$

Proof. Let P_6 be a triangular prism having 7 faces. Now we are going to find the local fractional metric dimension of triangular prism P_6. So, by definition of local fractional metric dimension, all possible resolving sets are given by

$$\begin{aligned}
\bullet\, \Re\{t_1^1, t_3^1\} &= V(P_6) - \{t_5^1, t_6^1\},\\
\bullet\, \Re\{t_1^1, t_2^1\} &= V(P_6) - \{t_5^1\},\\
\bullet\, \Re\{t_1^1, t_5^1\} &= V(P_6) - \{t_2^1, t_3^1\},\\
\bullet\, \Re\{t_2^1, t_5^1\} &= V(P_6) - \{t_1^1, t_4^1, t_6^1\},\\
\bullet\, \Re\{t_2^1, t_4^1\} &= V(P_6) - \{t_5^1, t_6^1\},\\
\bullet\, \Re\{t_2^1, t_6^1\} &= V(P_6) - \{t_3^1, t_4^1, t_5^1\},\\
\bullet\, \Re\{t_3^1, t_4^1\} &= V(P_6) - \{t_5^1\},\\
\bullet\, \Re\{t_3^1, t_5^1\} &= V(P_6) - \{t_1^1, t_4^1\},\\
\bullet\, \Re\{t_4^1, t_6^1\} &= V(P_6) - \{t_1^1, t_2^1, t_5^1\},\\
\bullet\, \Re\{t_5^1, t_6^1\} &= V(P_6) - \{t_3^1\},\\
\bullet\, \Re\{t_4^1, t_5^1\} &= V(\zeta^1(P_6) - \{t_2^1, t_3^1, t_6^1\}.
\end{aligned} \tag{17}$$

Since, all sets are the resolving sets but our interest is in least cardinality. Hence the minimum cardinality of resolving set is 3. So by relation, we have

$$ldim_f(P_6) = \frac{6}{3}. \tag{18}$$



P_6

$\square$

Proposition 7. *let P_7 be a Triangular prism constructed as a Halin graph from six vertex tree having seven faces. Then the local fractional metric dimension of P_7 is given by*

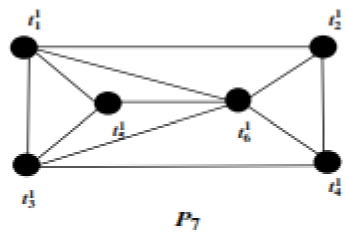
$$ldim_f(P_7) \quad = \quad 2. \tag{19}$$

Proof. Let P_7 be a triangular prism having 7 faces and let t_1^1,t_2^1,t_3^1,t_4^1,t_5^1,t_6^1 be the vertices of prism P_7. Now we are going to find the local fractional metric dimension of triangular prism P_7. So, by definition of local fractional metric dimension, all possible resolving sets are given by

$$\begin{aligned}
\bullet\, \Re\{t_1^1, t_3^1\} &= V(P_7) - \{t_5^1, t_6^1\}, \\
\bullet\, \Re\{t_1^1, t_2^1\} &= V(P_7) - \{t_6^1\}, \\
\bullet\, \Re\{t_1^1, t_5^1\} &= V(P_7) - \{t_3^1, t_4^1, t_6^1\}, \\
\bullet\, \Re\{t_1^1, t_6^1\} &= V(P_7) - \{t_2^1, t_3^1, t_5^1\}, \\
\bullet\, \Re\{t_2^1, t_4^1\} &= V(P_7) - \{t_5^1, t_6^1\}, \\
\bullet\, \Re\{t_2^1, t_6^1\} &= V(P_7) - \{t_1^1, t_4^1\}, \\
\bullet\, \Re\{t_3^1, t_4^1\} &= V(P_7) - \{t_6^1\}, \\
\bullet\, \Re\{t_3^1, t_5^1\} &= V(P_7) - \{t_1^1, t_2^1, t_6^1\}, \\
\bullet\, \Re\{t_4^1, t_6^1\} &= V(P_7) - \{t_2^1, t_3^1\}, \\
\bullet\, \Re\{t_5^1, t_6^1\} &= V(P_7) - \{t_3^1\}, \\
\bullet\, \Re\{t_3^1, t_6^1\} &= V(P_7) - \{t_1^1, t_4^1, t_5^1\}.
\end{aligned} \tag{20}$$

Since, all sets are the resolving sets but our interest is in least cardinality. Hence the minimum cardinality of resolving set is 3. So by relation, we have

$$ldim_f(P_7) \quad = \quad \frac{6}{3}. \tag{21}$$



□

Proposition 8. *let P_8 be a Triangular prism constructed as a Halin graph from six vertex tree having seven faces. Then the local fractional metric dimension of P_8 is given by*

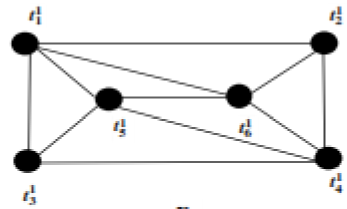
$$ldim_f(P_8) = 2. \tag{22}$$

Proof. Let P_8 be a triangular prism having 7 faces and let t_1^1,t_2^1,t_3^1,t_4^1,t_5^1,t_6^1 be the vertices of prism P_8. Now we are going to find the local fractional metric dimension of triangular prism P_8. So, by definition of local fractional metric dimension, all possible resolving sets are given by

$$\begin{aligned}
\bullet\, \Re\{t_1^1, t_3^1\} &= V(P_8) - \{t_5^1\},\\
\bullet\, \Re\{t_1^1, t_2^1\} &= V(P_8) - \{t_6^1\},\\
\bullet\, \Re\{t_1^1, t_5^1\} &= V(P_8) - \{t_3^1, t_6^1\},\\
\bullet\, \Re\{t_1^1, t_6^1\} &= V(P_8) - \{t_3^1, t_4^1\},\\
\bullet\, \Re\{t_2^1, t_4^1\} &= V(P_8) - \{t_6^1\},\\
\bullet\, \Re\{t_2^1, t_6^1\} &= V(P_8) - \{t_1^1, t_3^1, t_4^1\},\\
\bullet\, \Re\{t_3^1, t_4^1\} &= V(P_8) - \{t_5^1\},\\
\bullet\, \Re\{t_3^1, t_5^1\} &= V(P_8) - \{t_1^1, t_2^1, t_4^1\},\\
\bullet\, \Re\{t_4^1, t_6^1\} &= V(P_8) - \{t_2^1, t_5^1\},\\
\bullet\, \Re\{t_5^1, t_6^1\} &= V(P_8) - \{t_3^1\},\\
\bullet\, \Re\{t_4^1, t_5^1\} &= V(P_8) - \{t_1^1, t_2^1\}.
\end{aligned} \tag{23}$$

Since, all sets are the resolving sets but our interest is in least cardinality. Hence the minimum cardinality of resolving set is 3. So by relation, we have

$$ldim_f(P_8) = \frac{6}{3}. \tag{24}$$



P_8

□

Proposition 9. *let P_9 be a Triangular prism constructed as a Halin graph from six vertex tree having seven faces. Then the local fractional metric dimension of P_9 is given by*

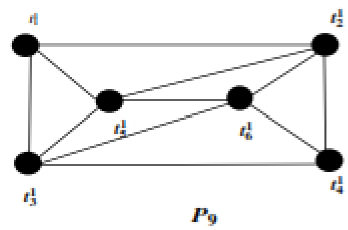
$$ldim_f(P_9) \quad = \quad 2. \tag{25}$$

Proof. Let P_9 be a triangular prism having 7 faces and let t_1^1,t_2^1,t_3^1,t_4^1,t_5^1,t_6^1 be the vertices of prism P_9. Now we are going to find the local fractional metric dimension of triangular prism P_9. So, by definition of local fractional metric dimension, all possible resolving sets are given by

$$\begin{aligned}
\bullet\, \Re\{t_1^1, t_3^1\} &= V(P_9) - \{t_5^1\},\\
\bullet\, \Re\{t_1^1, t_2^1\} &= V(P_9) - \{t_5^1\},\\
\bullet\, \Re\{t_1^1, t_5^1\} &= V(P_9) - \{t_2^1, t_3^1, t_4^1\},\\
\bullet\, \Re\{t_2^1, t_5^1\} &= V(P_9) - \{t_1^1, t_6^1\},\\
\bullet\, \Re\{t_2^1, t_4^1\} &= V(P_9) - \{t_6^1\},\\
\bullet\, \Re\{t_2^1, t_6^1\} &= V(P_9) - \{t_4^1, t_5^1\},\\
\bullet\, \Re\{t_3^1, t_4^1\} &= V(P_9) - \{t_6^1\},\\
\bullet\, \Re\{t_3^1, t_5^1\} &= V(P_9) - \{t_1^1, t_6^1\},\\
\bullet\, \Re\{t_4^1, t_6^1\} &= V(P_9) - \{t_1^1, t_2^1, t_3^1\},\\
\bullet\, \Re\{t_5^1, t_6^1\} &= V(P_9) - \{t_3^1\},\\
\bullet\, \Re\{t_3^1, t_6^1\} &= V(P_9) - \{t_4^1, t_5^1\}.
\end{aligned} \tag{26}$$

Since, all sets are the resolving sets but our interest is in least cardinality. Hence the minimum cardinality of resolving set is 3. So by relation, we have

$$ldim_f(P_9) \quad = \quad \frac{6}{3}. \tag{27}$$



Triangular prism with 5,6 and 7 faces

□

4 Rotationally symmetric triangular prism planar networks having five faces forming circular ladder network

Let $G_1, G_2, G_3, ..., G_k$ be k disjoint copies, with $k \geq 1$ of a triangular prism whose set of vertices is $V(G) = \{t_1, t_2, t_3, ..., t_n\}$. Let $t_i^k \in V(G_k)$ denote the corresponding copy of each vertex $V_i \in V(G)$. Then the total number of vertices of k copies are $4k + 2$. In this section, we find sequences of local fractional metric dimension of k disjoint copies of triangular prism network with 5 possible faces. Finally, we compute upper bound of local fractional metric dimension of k disjoint copies.

Proposition 10. *Let P_1 be a triangular prism constructed as a Halin graph from six vertex tree. Let $k \geq 1$ be a positive integer, then*

$$ldim_f(\zeta^k(P_1) \leq \begin{cases} \frac{3}{2}. & \text{if } i=1, \\ \frac{4k+2}{2k+2} & \text{if } i \text{ is even}, \\ \frac{4k+2}{2k+2} & \text{otherwise}. \end{cases} \tag{28}$$

Proof. Let P_1 be a triangular prism having 5 faces and let $t_1^1, t_2^1, t_3^1, t_4^1, t_5^1, t_6^1$ be the vertices of prism $(\zeta^1(P_1)$. Now we are going to find the upper bound sequences of triangular prism P_1 by connecting their k copies. First,we find the local fractional metric dimension of $(\zeta^1(P_1)$ and then we find local fractional metric dimension of second copy by connecting first copy with second copy. At the end, we generalize all sequences of all k copies that form a circular ladder. So, by definition of local fractional metric dimension, all possible resolving sets for $k = 1$ are given by

$$\begin{aligned}
\bullet\, \Re\{t_1^1, t_3^1\} &= V(\zeta^1(P_1)) - \{t_5^1, t_6^1\},\\
\bullet\, \Re\{t_1^1, t_2^1\} &= V(\zeta^1(P_1)) - \phi,\\
\bullet\, \Re\{t_1^1, t_5^1\} &= V(\zeta^1)(P_1) - \{t_3^1, t_4^1\},\\
\bullet\, \Re\{t_2^1, t_4^1\} &= V(\zeta^1)(P_1) - \{t_5^1, t_6^1\},\\
\bullet\, \Re\{t_2^1, t_6^1\} &= V(\zeta^1)(P_1) - \{t_3^1, t_4^1\},\\
\bullet\, \Re\{t_3^1, t_4^1\} &= V(\zeta^1)(P_1) - \phi,\\
\bullet\, \Re\{t_3^1, t_5^1\} &= V(\zeta^1)(P_1) - \{t_1^1, t_2^1\},\\
\bullet\, \Re\{t_4^1, t_6^1\} &= V(\zeta^1)(P_1) - \{t_1^1, t_2^1\},\\
\bullet\, \Re\{t_5^1, t_6^1\} &= V(\zeta^1)(P_1) - \phi.
\end{aligned} \tag{29}$$

Since, the minimum cardinality of resolving set is 4. So by relation, we have

$$ldim_f(\zeta^1(P_1) \leq \frac{6}{4}. \tag{30}$$

For $k = 2$, all possible resolving sets are given by,

$$\begin{aligned}
\bullet\, \Re\{t_1^1, t_3^1\} &= V(\zeta^2(P_1) - \{t_5^i, t_6^i\}, \text{ for } i = 1,2\\
\bullet\, \Re\{t_1^1, t_2^1\} &= V(\zeta^2(P_1) - \phi,\\
\bullet\, \Re\{t_1^1, t_5^1\} &= V(\zeta^2(P_1) - \{t_3^1, t_4^i\}, \text{ for } i = 1,2\\
\bullet\, \Re\{t_2^1, t_4^1\} &= V(\zeta^2(P_1) - \{t_5^i, t_6^i\}, \text{ for } i = 1,2\\
\bullet\, \Re\{t_2^1, t_6^1\} &= V(\zeta^2(P_1) - \{t_3^1, t_4^i\}, \text{ for } i = 1,2\\
\bullet\, \Re\{t_3^1, t_4^1\} &= V(\zeta^2(P_1) - \phi,\\
\bullet\, \Re\{t_3^1, t_5^1\} &= V(\zeta^2(P_1) - \{t_1^1, t_2^i\}, \text{ for } i = 1,2\\
\bullet\, \Re\{t_4^1, t_6^1\} &= V(\zeta^2(P_1) - \{t_1^1, t_2^i\}, \text{ for } i = 1,2\\
\bullet\, \Re\{t_5^1, t_6^1\} &= V(\zeta^2(P_1) - \phi,\\
\bullet\, \Re\{t_2^1, t_2^2\} &= V(\zeta^2(P_1) - \phi,\\
\bullet\, \Re\{t_2^1, t_5^2\} &= V(\zeta^2(P_1) - \{t_3^1, t_4^i\}, \text{ for } i = 1,2\\
\bullet\, \Re\{t_4^1, t_4^2\} &= V(\zeta^2(P_1) - \phi,\\
\bullet\, \Re\{t_4^1, t_5^2\} &= V(\zeta^2(P_1) - \{t_1^1, t_2^i\}, \text{ for } i = 1,2\\
\bullet\, \Re\{t_2^2, t_4^2\} &= V(\zeta^2(P_1) - \{t_5^i, t_6^i\}, \text{ for } i = 1,2\\
\bullet\, \Re\{t_2^2, t_6^2\} &= V(\zeta^2(P_1) - \{t_3^1, t_4^i\}, \text{ for } i = 1,2\\
\bullet\, \Re\{t_4^2, t_6^2\} &= V(\zeta^2(P_1) - \{t_1^1, t_2^i\}, \text{ for } i = 1,2\\
\bullet\, \Re\{t_5^2, t_6^2\} &= V(\zeta^2(P_1) - \phi.
\end{aligned} \tag{31}$$

Since, the minimum cardinality of resolving set is 6, So by relation, we have

$$ldim_f(\zeta^2(P_1) \leq \frac{10}{6}. \tag{32}$$

For $k \geq 3$. The symmetry of the rotational triangular P_1 prism enables us to focus on the following resolving neighbourhoods,

$$
\begin{aligned}
&\bullet \Re\{t_1^1, t_3^1\} = \Re\{t_2^i, t_4^i\} = V(\zeta^k(P_1) - \{t_5^i, t_6^i\} \text{ for } 1 \le i \le k \\
&\bullet \Re\{t_2^i, t_6^i\} = V(\zeta^k(P_1) - \{t_3^i, t_4^i\} \text{ for } 1 \le i \le k \\
&\bullet \Re\{t_2^i, t_5^{i+1}\} = V(\zeta^k(P_1) - \{t_3^1, t_4^i\} \text{ for } 1 \le i \le k \\
&\bullet \Re\{t_4^i, t_6^i\} = V(\zeta^k(P_1) - \{t_1^1, t_2^i\} \text{ for } 1 \le i \le k \\
&\bullet \Re\{t_4^i, t_5^{i+1}\} = V(\zeta^k(P_1) - \{t_1^1, t_2^i\} \text{ for } 1 \le i \le k \\
&\bullet \Re\{t_3^1, t_5^1\} = V(\zeta^k(P_1) - \{t_1^1, t_2^i\} \text{ for } 1 \le i \le k \\
&\bullet \Re\{t_2^i, t_2^{i+1}\} = V(\zeta^k(P_1) - \phi \text{ for } 1 \le i \le k \\
&\bullet \Re\{t_4^i, t_4^{i+1}\} = V(\zeta^k(P_1) - \phi \text{ for } 1 \le i \le k \\
&\bullet \Re\{t_5^i, t_6^i\} = V(\zeta^k(P_1) - \phi \text{ for } 1 \le i \le k \\
&\bullet \Re\{t_1^1, t_2^1\} = \Re\{t_3^1, t_4^1\} = V(\zeta^k(P_1) - \phi \text{ for } 1 \le i \le k \\
&\bullet \Re\{t_1^1, t_5^1\} = V(\zeta^k(P_1) - \{t_3^1, t_4^i\} \text{ for } 1 \le i \le k
\end{aligned} \tag{33}
$$

They have the following cardinalities.

$$
\begin{aligned}
&\bullet \mid \Re\{t_1^1, t_3^1\} \mid = \mid \Re\{t_2^i, t_4^i\} \mid = 2k + 2\text{for}1 \le i \le k, \\
&\bullet \mid \Re\{t_1^1, t_2^1\} \mid = \mid \Re\{t_3^1, t_4^1\} \mid = 4k + 2\text{for}1 \le i \le k, \\
&\bullet \mid \Re\{t_2^i, t_5^{i+1}\} \mid = 3k + 1 \text{ for}1 \le i \le k, \\
&\bullet \mid \Re\{t_1^1, t_5^1\} \mid = 3k + 1 \text{ for}1 \le i \le k, \\
&\bullet \mid \Re\{t_4^i, t_6^i\} \mid = 3k + 1 \text{ for } 1 \le i \le k, \\
&\bullet \mid \Re\{t_4^i, t_5^{i+1}\} \mid = 3k + 1 \text{ for } 1 \le i \le k, \\
&\bullet \mid \Re\{t_2^i, t_2^{i+1}\} \mid = 4k + 2 \text{ for } 1 \le i \le k, \\
&\bullet \mid \Re\{t_4^i, t_4^{i+1}\} \mid = 4k + 2 \text{ for } 1 \le i \le k, \\
&\bullet \mid \Re\{t_5^i, t_6^i\} \mid = 4k + 2 \; for \; 1 \le i \le k, \\
&\bullet \mid \Re\{t_2^i, t_5^{i+1}\} \mid = 3k + 1 \text{ for } 1 \le i \le k, \\
&\bullet \mid \Re\{t_2^i, t_6^i\} \mid = 3k + 1 \text{ for } 1 \le i \le k, \\
&\bullet \mid \Re\{t_3^1, t_5^1\} \mid = 3k + 1 \text{ for } 1 \le i \le k.
\end{aligned} \tag{34}
$$

As $2k + 2$ is the minimum cardinality.Hence, we have

$$ldim_f(\zeta^k(P_1) \le \frac{4k + 2}{2k + 2} \tag{35}$$

□

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