VFAST Transactions on Education and Social Sciences
http://vfast.org/journals/index.php/VTESS@ 2020 ISSN(e): 2309-3951;ISSN(p): 2411-0221
Volume 8, Number 1, January-December, 2020

pp. 47–64

THE EXTREME VALUE THEORY AS A RISK MODELING TOOL FOR THE NON-PERFORMING LOANS

Attia Rani, Faisal Mir*, Faisal Shehzad, Muhammad Shujahat
Department of Management Sciences, COMSATS University, Islamabad, Pakistan
*Corresponding author: mir@gmail.com

Abstract: The study proposes that the Non-Performing Loans (NPLs) is a rare event. Consequently, this study proposes that the classes of time series modeling and macroeconomic approaches are not appropriate for the risk modeling and assessment of NPLs. Consequently, this study proposes Extreme Value Theory (EVT) as an alternative tool for the risk modeling and assessment of NPL. The data of Asian countries for the 28 quarters (2010-2016) available on the World Bank website is accessed for testing the proposition with the use of analysis techniques. It was found that extreme value theory could be the most suitable tool for the risk assessment of non-performing loans. Moreover, the study ranks the Asian countries concerning the risk of NPLs. Finally, the study discusses the implications for the financial institutions for NPLs and policymakers.

Keywords: Non-performing Loan, Extreme Value Theory, ARMA, Risk Modeling, Risk Assessment, Risk Management

Introduction The banking sector is an important part of the financial system which plays a significant role in the growth of an economy. Stability of banking sector is a key for the development of an economy (e.g., Sontakke and Tiwari, 2013). In the banking industry, the loans are the greater portion of aggregate assets. These loans generate massive income in the form of interest for banks, and this income defines the financial performance of banks. Though, some of these loans commonly become non-performing loans and have an adverse impact on the bank's performance (e.g., Joseph *et al*, 2012).

A borrower must repay principal amount along with interest amount to service the loan. The operational definition of non-performing loan (NPL) for this study is : a loan which is not producing revenue and: (1) receiving of principal and interest amount in full is not to be expected, (2) principal amount or interest amount is outstanding for more than 90 days or (3) full payments have not been received after the date of maturity (Louangrath, 2015; Cucinelli, 2015).

Banking sector faces failure due to non-performing loans, and more often these loans become bad debts. Non-performing loan influences bank's power of lending and curtails the efficiency (Farooq *et al*, 2010; Aurangzeb, 2012). As banks play a significant role in the development of an economy, so it is essential to find out the problems that trouble the bank performance. Non-performing loans impede the banking sector performance at large and as a consequence is the cause of financial and banking crisis (e.g., Joseph *et al*, 2012; Cucinelli, 2015; Shah *et al*, 2016).

Due to these reasons, it is very crucial to control the non-performing loan for development of an economy. Otherwise, the capital can be jammed into non-profitable projects and businesses. This not only harms the stability of financial sector but also the economic development of the country. So to deal with the problem of non-performing loans, it is important to use appropriate risk assessment tool for these loans in the financial institutions.

The probability of non-performing loan is a type of credit risk which arises when the borrower is unable to pay the required loan. In the literature two methods; pure time series and macroeconomic approaches are used for the assessment of NPLs (e.g., Louangrath, 2015; Messai, 2013; Makri *et al*, 2014; Ghosh, 2015). Time series modeling is used to estimate upcoming values of the series. A time series model that records a single variable is termed as univariate. Univariate time series modeling includes Autoregressive (AR), Moving average (MA) and Auto regressive integrated moving average (ARIMA). When time series model records more than one variable, then the model is termed as multivariate. Multivariate times series modeling include the autoregressive integrated moving average (ARIMA) (Louangrath, 2015).

The second approach is based on macroeconomic models, which considers factors that affect NPL loans and bank performance. The main macroeconomic factor is the Gross domestic product (GDP). Growth in real GDP increases the ability of borrowers to refund their debts and should contribute to a lower NPL ratio (Greenidge and Grosvenor, 2010).

However, literature poses that AR, MA and ARIMA class of time series models are not fit to model the rare events as these models provide a good fit for the normal phenomenon. While NPL is a rare or extreme event over the maturity time of loans, therefore, AR class is not appropriate to model the rare event for risk assessment. In the same line, macroeconomic approach models the normal economic situations where the assumption of normal distribution is taken. But NPLs are rare or extreme events. For these events, one should consider extreme value distribution or the distribution with a fat tail that can handle extreme values (Taleb, 2007; McNeil et al., 2015). How then could the limitations of the AR class and macroeconomic modeling approaches be addressed?

This study proposes that the extreme value theory (EVT) as an alternative to AR class and macroeconomic models for risk assessment of NPL. As NPL is a rare event, therefore, EVT is suitable to model the risk assessment of NPL. EVT focuses on the extreme and rare events or extreme tail losses (Charras-garrido et al., 2013). Hence, the EVT models could serve at least two purposes: first these models are better to capture extreme events; and second, it gives the better forecast regarding longer period.

To the best of authors' knowledge and literature review, there is one study, (Louangrath, 2015) that has proposed and tested EVT as an alternative to AR class for NPL risk assessment. This paper attempted to model NPL using EVT theory based on only eight quarters data (2013-2014) of eight industries for Thailand. The study ranked the industries concerning risk level. However, this study of Louangrath (2015) has some limitations:

1. This study claims that EVT is a short-term method. However, the literature posits that EVT provides long-term forecast in a better way (McNeil *et al*, 2015).
2. The study used the data of eight industries from just one country and only eight quarters data. However, EVT is a time series method which needs quite a few observations, spanned over relatively more quarters.
3. This study assumes that EVT is a non-parametric method while the literature claims that EVT is a semi-parametric time series approach (McNeil *et al,* 2015).
4. It seems that above mentioned three limitations are because that this study is the conference presented study that was not published in a quality outlet (e.g., the journal that follows the blind review process).

Therefore, from the above limitations, it is deduced that there should be a study which overcomes these limitations. Consequently, the purpose of this study is to propose EVT as an alternative to AR class and other tools like macro-econometric modeling, for risk assessment of NPLs. The study tests the proposition using the data of 26 Asian countries available on the World Bank website from 2010 to 2016 on NPLs for 28 quarters.

The remaining of the paper is organized as follows. Section 2 covers the methodology. Section 3 presents the results to test the propositions of the study. Finally, section 4 concludes the study.

2 Methodology

2.1 Data

This is a quantitative study to find out the suitable tools to model the NPL ratios. NPL ratio, which is unit of analysis in this research, is equal to bank non-performing loan to total gross loan. To attain the objectives of the research, time series data of non-performing loans (NPL) of Asian banking sector is used.

There are 49 numbers of countries in Asia, and complete data of 26 numbers of countries was available on the World Bank website. Therefore, this study used NPLs ratio of 26 numbers of Asian countries from 2010 to 2016 for 28 quarters.

2.2 Analysis techniques

The study used the EVT time series modeling. ARMA-GARCH modeling is necessary to filter the NPL data before applying EVT to capture the autocorrelation effect from the NPL data. Only then the EVT is to apply on the filter data. Therefore, this study finds imperative to discuss and apply ARMA and GARCH before EVT.

2.2.1 AR model

In autoregressive modeling (AR), a time series model, financial variables are predicted by using their past information and also include the previous and current value of an error term (Brooks, 2014), the current value of variable is based on its previous values and an error term, So that:

$$y_t = \mu+\varphi_1 y_{t-1} +\varphi_2 y_{t-2} +\cdots+\varphi_p y_{t-p} +ut$$

P is the order of AR model, and μ is known as white noise disturbance term.

2.2.2 MA models: The second type of time series modeling is moving average (MA). The current value of the variable is based on the current and previous value of the random variable which is independently and identically distributed. It is the linear combination of white noise error term.

$$y_t = \mu+u_t +\theta_1 u_{t-1} +\theta_2 u_{t-2} +\cdots+\theta_q u_{t-q}$$

Where q is the order of MA and μ is the process mean (Brooks, 2014).

In AR model the series *Yt* is regressed against its previous period values and in MA *Yt* is regressed against its current and prior period white noise disturbance term. Forecasting the error term under the MA model is difficult as compared to AR Model. In MA model it is not readily visible, but in AR Model it is calculated through standard least square technique. There is a requirement of a suitable sample size to find the white noise term for the equation in first MA model. Then afterward more data points are included, and the alteration in the error term is recorded for the study of its distribution. So this requires larger sample size, but in NPL analysis, it is not feasible as NPLs problem needs quick intervention to avoid further losses (Masood *et al*, 2010).

2.2.3 ARMA

The third method is combining the AR and MA and getting ARMA Model that is an autoregressive moving average model. It describes that the current value of the variable is based on its past value and liner combination white noise disturbance term (current and previous value).

$y_t = \mu + \varphi_1 y_{t-1} + \varphi_2 y_{t-2} + \cdots + \varphi_p y_{t-p} + \theta_1 u_{t-1} + \theta_2 u_{t-2} + \cdots + \theta_q u_{t-q} + ut$ (Brooks, 2014)

Due to external shocks, the time series not reverts to its mean and established a new mean. In this case, autoregressive integrated moving average model (ARMIA) is used. So to capture external shock ARMA model becomes ARIMA as given below:

$$X_t = \mu + \phi_1 X_{t-1} + \phi_2 X_{t-2} + \dots + \phi_p X_{t-p} - \theta_1 u_{t-1} - \theta_2 u_{t-2} - \dots - \theta_q u_{t-q} - u_t$$

2.2.4 Generalized ARCH (GARCH) Models

The GARCH model is used to capture heteroscedastic effect in the given series before the application of EVT. The GARCH model was established independently by Bollerslev (1986) and Taylor (1986). The GARCH model permits the conditional variance to be reliant upon own previous lags so that the conditional variance equation in the simplest case is

$$\sigma_t^2 = \alpha_0 + \alpha_1 u_{t-1}^2 + \beta \sigma_{t-1}^2$$

This is a GARCH (1, 1) model. σ^2t is called conditional variance, and it is a one-period onward approximation for the variance calculated by any past information supposed to be relevant. Interpretation of currently fitted variance, ht can be made by using the GARCH model ht is a weighted function of a long-term average value (dependent on α_0), evidence about volatility in the preceding period ($\alpha_1 u^2_{t-1}$) and the fitted variance of the model in the preceding period ($\beta\sigma t-1^2$). The GARCH model can be stated in a form that displays that it is effectively an ARMA model used for the conditional variance.

2.2.4 EVT model

Extreme value theory is used to forecast the occurrence of rare events. It is a statistical branch to treat the extreme deviations from the median of probability distributions (Charras-garrido et al, 2013). There are two ways to categorize extreme values in the data. If we take a random variable X showing returns or losses:

In first method, we take the maximum value of variables in consecutive periods such as monthly or yearly. These values are the extreme events and named as block maxima. In following Figure 1, the values *X*2, *X*5, *X*7 and, *X*11 signify the block maxima in four periods and each period contain three observations (Gilli and kellezi, 2006)

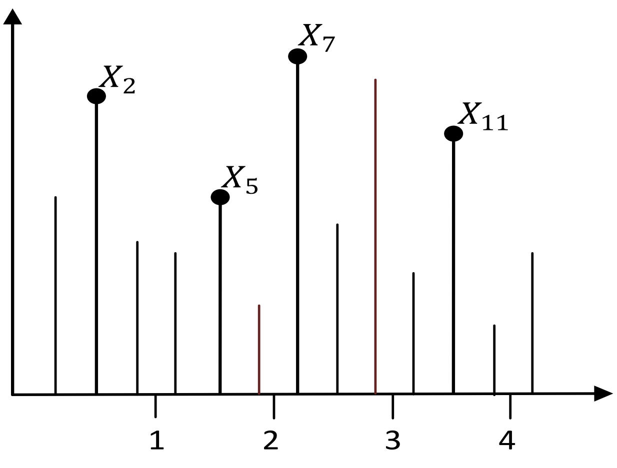

Figure 1: Block maxima method

In this series under specific form converges to Gumbel, Frechet or Weibull distribution. Together these three distributions are called generalized extreme value (GEV) distribution.

In the second method, which is Peak over Threshold (POT), we focus on the values exceeding a given threshold. In following figure 2, values $X1$, $X2$, $X7$, $X8$, $X9$ and $X11$ exceed the threshold u and create extreme events (Gilli and kellezi, 2006).

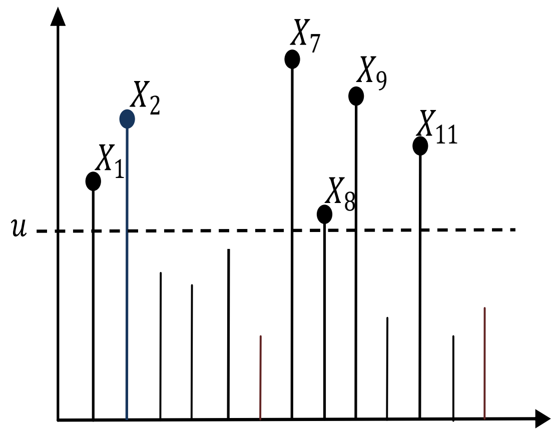

Figure 2: The Peak over Threshold method

The distribution of these excess values follows generalized Pareto distribution (GDP) for the high threshold. EVT is used for the estimation of high quantile say for 99.9% and above.
The method of block maxima is the old one and used for the data having seasonality. The second method which is threshold method is more efficient in analyzing the data so in current researches it is mostly used (Gilli and kellezi, 2006).

2.2.4.1 GEV Distribution

*I*f X_i shows the random variable sequence X_1……..X n where X is independently and identically distributed then maxima X n=max(X_1……..X n) joins to

$$H_{\xi,\mu,\sigma}(x) = \begin{cases} e^{-\left(1+\xi\frac{(x-\mu)}{\sigma}\right)^{\frac{-1}{\xi}}} & if \xi \neq 0 \\ e^{-e^{\frac{-(x-\mu)}{\sigma}}} & if \xi = 0 \end{cases}$$

$$While\ 1 + \xi\frac{(x-\mu)}{\sigma} > 0$$

This shows that maxima asymptotic distribution always links with GEV distribution whatever the original distribution would be. In above equation ξ is tail index and showing the thickness of tail, μ is the scalar parameter, and σ is tendency parameter.
In EVT the sample is divided into subsample, and minimum and maxima out of each sample are collected. Limiting distribution available of these extrema will become standard GEV distribution. In the initial step of analysis in extreme value theory first, we must verify the data distribution. As per Burnham and Anderson (2002) verification of data, distribution is needed as the suitable statistical test may be chosen with known distribution type for hypothesis testing and selection of model. In this research, we have twenty-six numbers of Asian countries with twenty-eight operating quarters between 2010 and 2016. The mean values for twenty-eight quarters of the twenty-six numbers of countries are used as the observed data set.

3 Result
The results are organized around diagnostic test, ARMA model, GARCH model and EVT models respectively. The diagnostic tests used in this study are for checking stationarity and arch effect. ARMA model is used to capture

autocorrelation effect out of NPL series. Then GARCH model is applied to remove the heteroskedastic effect out of the NPL series. After the due filtration of the NPL residual series, EVT model is deployed for risk assessment of NPL. The descriptive statistics of 26 numbers of countries are given in Table 1. Table 2 shows the Jarque-Bera results that explain that the series either follows the normal distribution or not.

Table 1 Descriptive Statistics

Country	Mean	Standard Deviation	Min	Max	Skewness	Kurtosis
ggregate NPL	-4.59e-05	0.036439	-0.113012	.053713	-1.373622	5.764034
Afghanistan	0.783894	2.729779	-7.196875	0.01562	0.640897	8.570195
Arab Emirate	-0.002702	0.026030	-0.057226	.054344	-0.249719	3.473904
Armenia	0.000197	0.064513	-0.213006	.112575	-0.922595	6.324689
Bhutan	-0.000586	0.101375	-0.234213	.325920	0.741302	6.357664
unei Darussalam	0.016250	0.910482	-1.515625	.637500	2.181490	11.70199
China	-0.001196	0.078750	-0.289936	.148864	-1.506627	9.018015
Cambodia	-0.000133	0.082862	-0.162750	.285719	1.199442	7.310822
Cyprus	-0.001907	0.096117	-0.321225	.210084	-0.847695	6.990179
Georgia	0.000260	0.062909	-0.126482	.218172	1.248352	7.464577
Hong Kong	-7.92e-05	0.039458	-0.130104	.079076	-0.825547	6.592218
Indonesia	6.45e-05	0.036672	-0.121378	.067135	-0.885001	6.541059
India	-0.000514	0.043870	-0.091371	.147232	0.957604	6.952999
Israel	-0.000741	0.016792	-0.043836	.026910	-0.873122	3.636553
Jordan	-0.000661	0.016622	-0.044877	.038088	-0.442766	4.234216
Japan	0.000244	0.020428	-0.031622	.070199	1.201640	6.978015
Kazakhstan	0.001210	0.068288	-0.161702	.221591	0.708024	6.636713
Kuwait	-0.000292	0.033845	-0.119431	.058721	-1.368335	7.629525
Lebanon	-0.000160	0.014756	-0.038458	.037501	-0.330006	4.830413
Malaysia	0.000224	0.032984	-0.100309	.081878	-0.386351	5.882664
Pakistan	-0.002564	0.014720	-0.047468	.033692	-0.520238	5.458428
Philippines	0.000155	0.059328	-0.195804	.150939	-0.714065	7.406055
ssian Federation	0.002977	0.013167	-0.042294	.037697	-0.876412	8.001235

Saudi Arabia	-0.000205	0.034214	-0.132810	.055983	-2.108321	10.51963
Singapore	-4.38e-05	0.029528	-0.072194	.090684	0.396418	6.013916
Thailand	-0.000219	0.027335	-0.090800	.049846	-1.297178	6.526574
Uzbekistan	6.28e-05	0.022772	-0.095821	.040606	-2.707795	14.48373

Table 2 Distribution Type under Jarque-Bera Test

Country	Jarque-Bera	Probability	Distribution
Aggregate NPL	15.82004	0.000367	Normal
Afghanistan	35.39257	0.000000	Normal
Arab Emirate	0.513525	0.773552	Not-Normal
Armenia	15.06072	0.000537	Normal
Bhutan	14.03336	0.000897	Normal
Brunei Darussalam	98.70860	0.000000	Normal
China	47.18355	0.000000	Normal
Cambodia	25.35191	0.000003	Normal
Cyprus	19.57904	0.000056	Normal
Georgia	27.25623	0.000001	Normal
Hong Kong	16.28140	0.000291	Normal
Indonesia	16.32501	0.000285	Normal
India	20.09814	0.000043	Normal
Israel	3.742447	0.153935	Not-Normal
Jordan	2.499744	0.286541	Not-Normal
Japan	22.50037	0.000013	Normal
Kazakhstan	15.86550	0.000359	Normal
Kuwait	30.12694	0.000000	Normal
Lebanon	3.943780	0.139194	Not-Normal
Malaysia	9.277937	0.009668	Not-Normal
Pakistan	7.720331	0.021065	Not-Normal
Philippines	22.34675	0.000014	Normal
Russian Federation	30.42513	0.000000	Normal
Saudi Arabia	77.42185	0.000000	Normal
Singapore	10.11695	0.006355	Not-Normal
Thailand	19.96604	0.000046	Normal
Uzbekistan	167.9216	0.000000	Normal

3.1 Diagnostic Tests

In data, there are three types of issues including non-stationary, serial correlation and heteroscedasticity. Augmented Dickey-Fuller (ADF), Kwiatkowski–Phillips–Schmidt–Shin (KPSS) test and Phillips-Perron (PP) test is used for non-stationary data. Ljung-Box Q-statistics is used for high-order serial correlation.

Our study utilized quarterly data of NPLs ratio for the period 2010 to 2016. Overall NPLs ratio of Asian countries specifies a general descending trend over the sample period as illustrated in Figure 3.

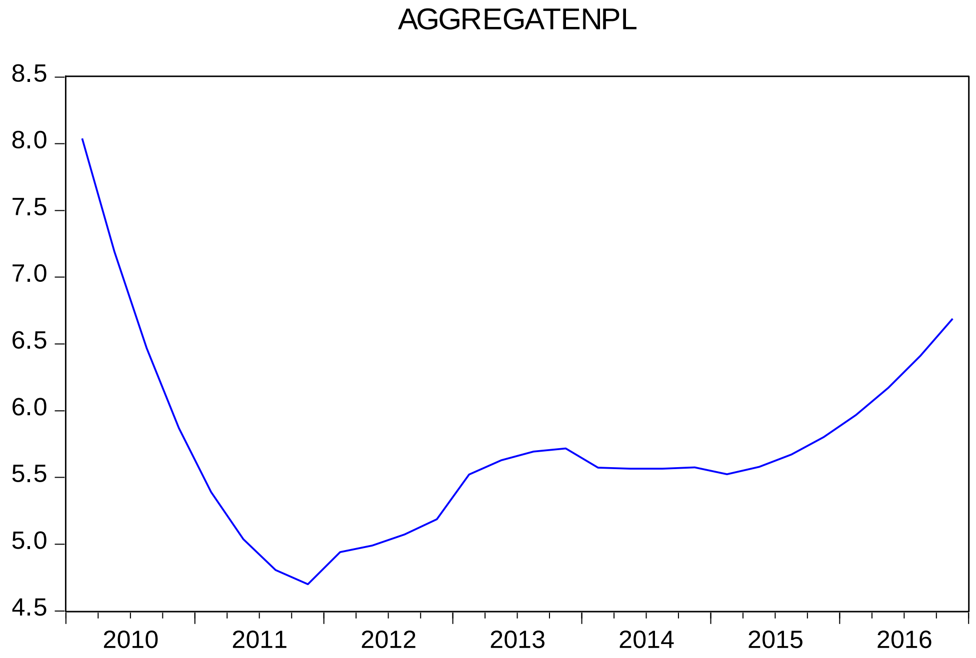

Figure 3: The plot of aggregated NPL series.

In the preliminary step of analysis diagnostic tests are used to find out the issues or problems in data. As in Table 3, ADF and PP test results show that NPL ratio of all the countries is difference stationarity at 5% and 1% levels and unit root tests with a trend and constant.

Table 3: Result of Stationarity Test

NPL Ratio	ADF	KPSS	Phillips Perron
Aggregate NPL	-5.928027**	0.208961*	-5.928027**
Afghanistan	-37.65676 **	0.168721 *	-6.181706 **
Arab Emirate	-6.301126 **	0.184881 *	-6.315864**
Armenia	-6.318554 **	0.280303 **	-6.318554 **
Bhutan	-5.819671 **	0.171102 *	0.5819671 **
Brunei Darussalam	-6.112335 **	0.400625**	-6.112335 **
China	-6.277187 **	0.360781 **	-6.277187 **
Cambodia	-5.902488 **	0.50 **	-5.902488 **
Cyprus	-6.525640 **	0.240061**	-6.525640 **
Georgia	-6.367990 **	0.299229 **	-6.394025 **
Hong Kong	-5.567478 **	0.50 **	-5.564706 **
Indonesia	-6.109482 **	0.281478 **	-6.109482 **
India	-6.199093 **	0.339621 **	-6.199093 **
Israel	-6.115349 **	0.189485 *	-6.115349 **
Jordan	-6.526167 **	0.186371 *	-6.560332 **
Japan	-5.617672 **	0.50 **	-5.616016 **

Kazakhstan	-5.834646 **	0.340980 **	-5.832984 **
Kuwait	-5.612975 **	0.292365 **	-5.609801 **
Lebanon	-6.123976 **	0.50 **	-6.123976 **
Malaysia	-6.597927 **	0.267719 **	-6.310444 **
Pakistan	-3.776753**	0.169346 *	-6.270719 **
Philippines	-6.273568 **	0.262041**	-6.273568 **
Russian Federation	-6.130251 **	0.165028 *	-6.147830 **
Saudi Arabia	-5.919275 **	0.50 **	-5.919275 **
Singapore	-6.087865 **	0.50 **	-6.087865 **
Thailand	-6.228239 **	0.315999**	-6.250029**
Uzbekistan	-5.723704 **	0.50 **	-5.723704 **

Q statistic is used to check the serial auto correlation in the data. P values less than 0.05 shows that there is a serial correlation in the data. The results of the test in Table 4 show that there is no serial correlation for data of Afghanistan, Israel, and Pakistan.

Table 4 Result of Serial Correlation Test

NPLs Ratio	Q-stats				P-Value			
	Lags 3	Lags 6	Lags 9	ags 12	Lags 3	Lags 6	Lags 9	ags 12
Aggregate NPL	.196	.734	.070	.765	007	015	003	008
Afghanistan	4864	5487	1537	2518	685	949	989	999
Arab Emirate	7828	3166	.835	.496	619	888	223	334
Armenia	.451	.491	.531	.106	002	000	001	005
Bhutan	.812	.672	.944	.554	008	023	009	001
Brunei Darussalam	.183	.760	.137	.353	007	010	047	137
China	.515	.578	.184	.870	006	005	017	052
Cambodia	.247	.629	.208	.527	010	071	033	101
Cyprus	.085	.202	.058	.080	001	000	000	001
Georgia	.231	.400	.438	.882	003	000	000	000
Hong Kong	.130	.610	.855	.939	011	050	019	068
Indonesia	5.050	2.638	3.190	3.417	.002	.000	.000	.001
India	2.985	2.061	5.288	7.158	.005	.001	.003	.007
Israel	.8464	.0940	.8066	.5598	.605	.797	.759	.819
Jordan	.9320	.1612	5.659	5.797	.402	.165	.074	.201
Japan	1.379	1.530	2.165	2.943	.010	.073	.008	.028
Kazakhstan	2.842	8.610	8.615	8.697	.005	.005	.029	.096
Kuwait	2.238	5.115	1.856	0.663	.007	.019	.009	.002
Lebanon	1.384	2.650	5.470	5.839	.010	.049	.079	.199
Malaysia	6.476	9.499	6.281	7.006	.001	.000	.000	.000

Pakistan	.4565	2.0559	4.0696	5.9044	0.692	0.914	0.907	0.921
Philippines	5.512	32.911	34.285	34.304	0.001	0.000	0.000	0.001
Russian Federation	.6332	.9579	2.6110	4.7958	0.652	0.924	0.978	0.964
Saudi Arabia	1.409	3.630	7.107	21.369	0.010	0.034	0.047	0.045
Singapore	1.602	5.102	27.600	30.797	0.009	0.019	0.001	0.002
Thailand	3.330	23.354	28.754	29.524	0.004	0.001	0.001	0.003
Uzbekistan	1.035	1.036	1.036	1.036	0.012	0.087	0.273	0.526

There is some serial correlation in data of Arab Emirate, Jordan and Russian Federation and Uzbekistan although q-test is not showing it, and remaining countries data have the serial correlation. For the serial correlation, ARMA test is applied.

3.2 ARMA model

The purpose of application of ARMA Models for 23 countries' NPL is to make the comparison of different modeling efficiency in time series approach and that of system analysis applied in this research. It regresses the current series against its past value. This method produces the higher level of correlations and higher coefficient of determination as shown in Table 5. A country for which R-square score is quite low is due to the reason that the countries do not have the uniform distribution

Table 5 ARMA Model with corresponding R- Square

NPLRatio	ARMA Model	R2
Aggregate NPL	$-0.000397-0.114924\, r_{t-1}+0.404229\, r_{t-4} - 0.694421\, r_{t-8} - \varepsilon_{t-1}$ (0.0005) (0.1339) (0.1599) (0.1192) (9391)	0.81
Arab Emirate	$-0.003048 -0.109102\, r_{t-1}-0.454127\, r_{t-8}+0.011668\, \varepsilon_{t-1}$ (0.0039) (0.4682) (0.1724) (0.5366)	0.27
Armenia	$00377-0.002313\, r_{t-1}-0.704915\, r_{t-4}-0.440395\, r_{t-8}-1.258450\, \varepsilon_{t-1}+0.258450\varepsilon_{t-2}$ (0.0012) (0.4289) (0.2720) (0.4487) (1144) (373.20)	0.80
Bhutan	$-0.000509 -0.016345\, r_{t-1}-0.212759\, r_{t-4} -0.419581\, r_{t-8} -0.832627\, r_{t-12} - 0.998541\, \varepsilon_{t-1}$ (0.0008) (0.0594) (0.2464) (0.2129)(0.2408) (55.23)	0.95
Brunei Darussalam	$0.017278 -.200857\, r_{t-1} -0.226297\, r_{t-4} - \varepsilon_{t-1}$ (0.0273) (0.1969) (0.2480) (5858)	0.65
China	$-0.000909 -0.084790\, r_{t-1} -0.539511\, r_{t-4} -0.654831\, r_{t-8} -1.123058\, \varepsilon_{t-1} +0.123058\, \varepsilon_{t-2}$ (0.0012) (0.6591) (0.2753) (0.4612) (902.87) (141.3)	0.80

Cambodia	0.000577 -0.187871 r_{t-1} -0.289679 r_{t-8} $-\varepsilon_{t-1}$ (0.0018) (0.2246) (0.2755) (0.6767)	0.66
Cyprus	-0.001039 -0.099385 r_{t-1} -0.755778r_{t-4}-0.396843r_{t-8} -1.062651 ε_{t-1} +0.062651ε_{t-2} (0.0015) (0.3510) (0.2697) (0.2818) (2021) (168.7)	0.81
Georgia	0.000682 +0.285900 r_{t-1} -0.516383 r_{t-4} -1.999997 ε_{t-1} +0.999997ε_{t-2} (0.0003) (0.3003) (0.1333) (14.07) (14.07)	0.83
Hong Kong	0.000276 -0.130695 r_{t-1} -0.471178 r_{t-8} $-\varepsilon_{t-1}$ (0.0008) (0.2221) (0.2588) (6259)	0.68
Indonesia	-6.20e-05 -0.104666 r_{t-1} -0.808380 r_{t-4} -0.446551 r_{t-8} $-\varepsilon_{t-1}$ (0.0007)(0.2506) (0.1982) (0.4503) (1215)	0.82
India	25e-05 +0.110714r_{t-1} -1.189655r_{t-4} -1.69823 r_{t-8} -0.796368 r_{t-12} -1.965782ε_{t-1}+0.987898 ε_{t-2} (0.0002) (0.1589) (0.1854) (0.1491) (0.2449) (120.2) (119.6)	0.94
Jordan	-0.000677 -0.215844r_{t-1} -0.339578r_{t-4}+0.165987ε_{t-1} (0.0035) (0.6874) (0.2200) (0.7516)	0.16
Japan	0.000292 -0.121045r_{t-1} -0.369551r_{t-8} -0.999997ε_{t-1} (0.0006) (0.3873) (0.1930) (2029)	0.66
Kazakhstan	0.001575 -0.226855r_{t-1} -0.244063r_{t-4} -0.781018 ε_{t-1} (0.0021) (0.2621) (0.1280) (0.2034)	0.59
Kuwait	3e-05 -0.020031r_{t-1} +0.453009 r_{t-4} -0.850293 r_{t-8} -1.171755 ε_{t-1} +0.171755ε_{t-2} (0.0005) (0.3542) (0.1686) (0.1815)(915.4) (194.3)	0.87
Lebanon	-0.000272 -0.222740r_{t-1} $-\varepsilon_{t-1}$ (0.0004) (0.2281) (5773)	0.62
Malaysia	-0.000279 -0.040124 r_{t-1} -0.609968r_{t-4}-1.89778 $-\varepsilon_{t-1}$ -1.89778-ε_{t-2} (0.0006) (0.5991) (0.1975) (1423) (330.7)	0.81
Philippines	-0.000127 -0.010592 r_{t-1} -0.745018r_{t-4}-0.324247r_{t-8} -1.220967ε_{t-1} +0.220967ε_{t-2} (0.0006) (0.7290) (0.3263) (0.3629) (1453) (412.8)	0.81
Russian Federation	-0.003010 -0.122374 r_{t-1} (0.0025) (0.1480)	0.015
Saudi Arabia	-4.62e-05 -0.173642r_{t-1} -0.499908r_{t-8} -0.999992ε_{t-1} (0.0011) (0.2029) (0.6130) (91781)	0.69

Singapore	$0.000257 +0.182889\, r_{t-1} -0.505052 r_{t-4} -0.748609 r_{t-8} -1.999990\varepsilon_{t-1}$ $+0.999990\varepsilon_{t-2}$ (0.0001) (0.1451) (0.1408) (0.1127) (21.879) (21.878)	0.90
Thailand	$-0.000487 +0.001139\, r_{t-1} -0.409667 r_{t-4} -1.255397\, \varepsilon_{t-1} +0.255397\varepsilon_{t-2}$ (0.0004) (0.5123) (0.1758) (630.41) (203.36)	0.71
Uzbekistan	$-8.8e-05 -0.156081 r_{t-1} - \varepsilon_{t-1}$ (0.0015) (0.2739) (8665)	0.59

Heteroscedasticity is checked through the arch test. If probability value is greater than 0.05, then there is no arch effect. Arch test for all the countries shows no arch effect in data, but the visual test shows the arch effect in data, so GARCH model is applied to make data more reliable (Table 6).

Table 6 GARCH Model with corresponding R- Square

NPL Ratio	GARCH Model	R^2
Afghanistan	$-1.84e-09+7.016381u^2_{t-1}$ (2.9e-06) (3.959)	-0.102
Arab Emirate	$0.000164+ 0.942815\, u^2_{t-1}$ (9.42e-05) (0.6702)	0.236
Armenia	$3.98e-05-0.044328\, u^2_{t-1}-0.227695\, \sigma^2_{t-1}$ (2.58e-05) (0.2564) (0.1158)	0.986
Bhutan	$0.003483-0.084182\, \sigma^2_{t-1}$ (0.0085) (2.6720)	0.749
Brunei Darussalam	$0.000116+0.445932\, u^2_{t-1}+0.936390\, \sigma^2_{t-1}$ (0.0009) (0.4136) (0.4529)	0.896
China	$0.002202-0.073476\, \sigma^2_{t-1}$ (0.004491) (2.227)	0.614
Cambodia	$-3.76e-12+ 2.785561\, u^2_{t-1}$ (1.49e-11) (1.030)	1.000
Cyprus	$1.89E-06+2.483153\, u^2_{t-1} -0.065861\, \sigma^2_{t-1}$ (1.13E-06) (1.4008) (0.1018)	0.996
Georgia	$1.63E-10+1.470508\, u^2_{t-1}$ (9.30E-11) (1.5547)	0.999
Hong Kong	$2.25E-08+3.161803\, u^2_{t-1}$ (1.15E-08) (2.3176)	0.999
Indonesia	$-1.34E-12+1.121238\, \sigma^2_{t-1}$ (1.73E-11) (0.1475)	1.000
India	$0.000420 -0.153473\, \sigma^2_{t-1}$ (0.0008) (2.4948)	0.828

Israel	$-1.91E-07 + 1.319993\, u^2_{t-1} + 0.296808\, \sigma^2_{t-1}$ (6.74E-08) (0.6426) (0.1397)	-0.495
Jordan	$5.47E-05 + 1.386924\, u^2_{t-1}$ (2.12E-05) (1.0963)	-0.255
Japan	$1.20E-13 + 1.324869\, u^2_{t-1}$ (6.60E-14) (1.5447)	1.000
Kazakhstan	$-4.91E-06 + 4.153285\, u^2_{t-1}$ (2.51E-05) (2.7287)	0.867
Kuwait	$0.000383 - 0.108435\, \sigma^2_{t-1}$ (0.0008) (2.3643)	0.779
Lebanon	$7.03E-09 + 1.291560\, u^2_{t-1} - 0.055229\, \sigma^2_{t-1}$ (4.69E-09) (1.0594) (0.4120)	0.999
Malaysia	$4.04E-10 + 11.13018\, u^2_{t-1}$ (1.10E-09) (4.3421)	0.999
Pakistan	$-2.43E-06 + 0.376349\, u^2_{t-1} + 0.552457\, \sigma^2_{t-1}$ (4.08E-07) (0.1070) (0.0643)	-0.169
Philippines	$-6.00E-11 + 13.24273\, u^2_{t-1}$ (8.48E-11) (1.6775)	0.999
Russian Federation	$0.000101 + 0.720542\, u^2_{t-1} - 0.415327\, \sigma^2_{t-1}$ (5.10E-05) (1.3974) (0.7814)	-0.051
Saudi Arabia	$0.000449 + 0.095750\, u^2_{t-1} + 0.470355\, \sigma^2_{t-1}$ (0.0007) (0.4056) (0.9058)	0.594
Singapore	$-9.32E-10 + 4.008051\, u^2_{t-1}$ (6.70E-09) (0.8520)	0.999
Thailand	$1.02E-07 - 0.120607\, u^2_{t-1} + 1.114972\, \sigma^2_{t-1}$ (8.82E-08) (0.0291) (0.0704)	0.997
Uzbekistan	$4.71E-08 + 2.046137\, u^2_{t-1}$ (2.57E-07) (0.6224)	0.578

3.1 Application of extreme value theory

Data of each country is filtered for outlier values by use of standard score method. After the findings of the outliers, the tail index is calculated to verify the type of distribution for each country data set of NPL. The Hill method (Hill, 1975) is used in calculating the tail index.

Table 7 Tail Index for NPL of each Country

Country	$Z \geq 1.65$	Tail Index(ε_j)	Distribution	Index Method
Afghanistan	Yes	-0.1798	Weibull	Hill' s Method
Arab Emirate	Yes	-0.3134	Weibull	Hill' s Method
Armenia	Yes	-0.4277	Weibull	Hill' s Method
Bhutan	Yes	-0.1535	Weibull	Hill' s Method
Brunei Darussalam	Yes	-0.0597	Weibull	Hill' s Method

China	No	-0.4207	Weibull	Hill’ s Method
Cambodia	No	-0.1065	Weibull	Hill’ s Method
Cyprus	Yes	-0.3461	Weibull	Hill’ s Method
Georgia	Yes	-0.1059	Weibull	Hill’ s Method
Hong Kong	No	-0.3717	Weibull	Hill’ s Method
Indonesia	No	-0.3955	Weibull	Hill’ s Method
India	Yes	-0.1299	Weibull	Hill’ s Method
Israel	Yes	-0.4997	Weibull	Hill’ s Method
Jordan	Yes	-0.3104	Weibull	Hill’ s Method
Japan	No	-0.0925	Weibull	Hill’ s Method
Kazakhstan	Yes	-0.1608	Weibull	Hill’ s Method
Kuwait	Yes	-0.4591	Weibull	Hill’ s Method
Lebanon	Yes	-0.2715	Weibull	Hill’ s Method
Malaysia	No	-0.2878	Weibull	Hill’ s Method
Pakistan	Yes	-0.3045	Weibull	Hill’ s Method
Philippines	No	-0.3002	Weibull	Hill’ s Method
Russian Federation	Yes	-0.2987	Weibull	Hill’ s Method
Saudi Arabia	No	-0.4991	Weibull	Hill’ s Method
Singapore	No	-0.1863	Weibull	Hill’ s Method
Thailand	Yes	-0.4404	Weibull	Hill’ s Method
Uzbekistan	No	-0.4680	Weibull	Hill’ s Method

The findings in Table 7 suggest that NPL ratios follow the Weibull distribution. The tail index is used to make the result more accurate by applying suitable analytical tools. Keeping in view the tail index, we can divide the data into different groups by distribution.

3.1.1 Weibull distribution analysis and the implication for NPL risk

The NPL ratios of all the countries under consideration are following the Weibull distribution. Weibull CDF is used to determine the risk level in the analysis of Weibull distribution. The direction of NPL trend is represented by beta. For $\beta>1$ shows that NPL increases with the passage of time, and $\beta<1$ shows that NPL will decrease concerning time. For $\beta=1$ means no change in the direction of trend. Results are summarized in Table 8 as below.

Table 8: Weibull Statistics for 26 Asian Countries

Country	a	b	beta=1/b	η=exp(a)	CDF	=1-CDF
Afghanistan	0.546	0.372	2.6901	1.727	1.000	0.000
United Arab Emirates	0.975	38.911	0.0257	2.652	0.998	0.002
Armenia	0.963	15.291	0.0654	2.619	0.966	0.035
Brunei Darussalam	0.481	1.413	0.7079	1.618	1.000	0.000

Bhutan	0.917	10.917	0.0916	2.501	0.996	0.004
China	0.951	12.225	0.0818	2.590	0.908	0.092
Cambodia	0.931	14.286	0.07	2.536	0.999	0.001
Cyprus	0.933	10.132	0.0987	2.541	0.992	0.008
Georgia	0.948	18.868	0.053	2.580	0.996	0.004
Hong Kong SAR, China	0.974	24.938	0.0401	2.649	0.993	0.007
Indonesia	0.977	26.810	0.0373	2.658	0.992	0.008
India	0.962	25.974	0.0385	2.617	1.000	0.000
Israel	0.989	58.140	0.0172	2.690	1.000	0.000
Jordan	0.986	59.880	0.0167	2.681	0.998	0.002
Japan	0.983	59.524	0.0168	2.672	0.999	0.001
Kazakhstan	0.947	15.974	0.0626	2.577	0.995	0.005
Kuwait	0.981	28.902	0.0346	2.666	0.994	0.006
Lebanon	0.988	68.493	0.0146	2.687	0.999	0.001
Malaysia	0.976	30.211	0.0331	2.654	0.999	0.001
Pakistan	0.984	66.225	0.0151	2.674	1.000	0.000
Philippines	0.958	16.260	0.0615	2.606	1.000	0.000
Russian Federation	0.996	70.922	0.0141	2.708	0.999	0.001
Saudi Arabia	0.982	28.571	0.035	2.670	0.999	0.001
Singapore	0.976	35.971	0.0278	2.653	0.999	0.001
Thailand	0.984	35.842	0.0279	2.675	0.995	0.005
Uzbekistan	0.988	41.841	0.0239	2.686	1.000	0.000

Only Afghanistan has increasing NPL trend concerning time. Value of β□□1 for all remaining 25 number of countries showing NPL trend is decreasing concerning time. From the risk management point of view, the two countries with β> 1 should be taken for intervention measures.

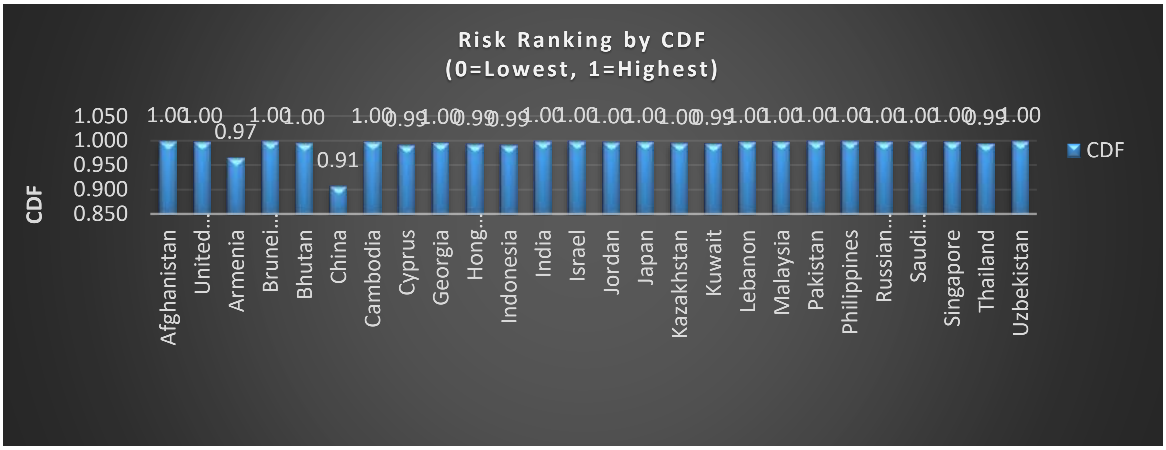

Figure 4: *Risk Ranking by Weibull's CDF*

NPL failure rate is represented by the value of the eta (η). The value of η is usually read with the CDF value. The corresponding failure rate is represented by CDF at the value of η. The Seven number of countries have CDF=1 at a value of η. Recall that trend of NPL for these countries are decreasing concerning time except two countries Afghanistan and Brunei Darussalam. The positive value of beta for all countries shows that its NPL risk is actual and may continue to increase. These finding must be read with the system reliability represented by R. Higher the value of R shows more reliability of the system to produce NPL. In this case, all the countries have a minimum value of R or system reliability.

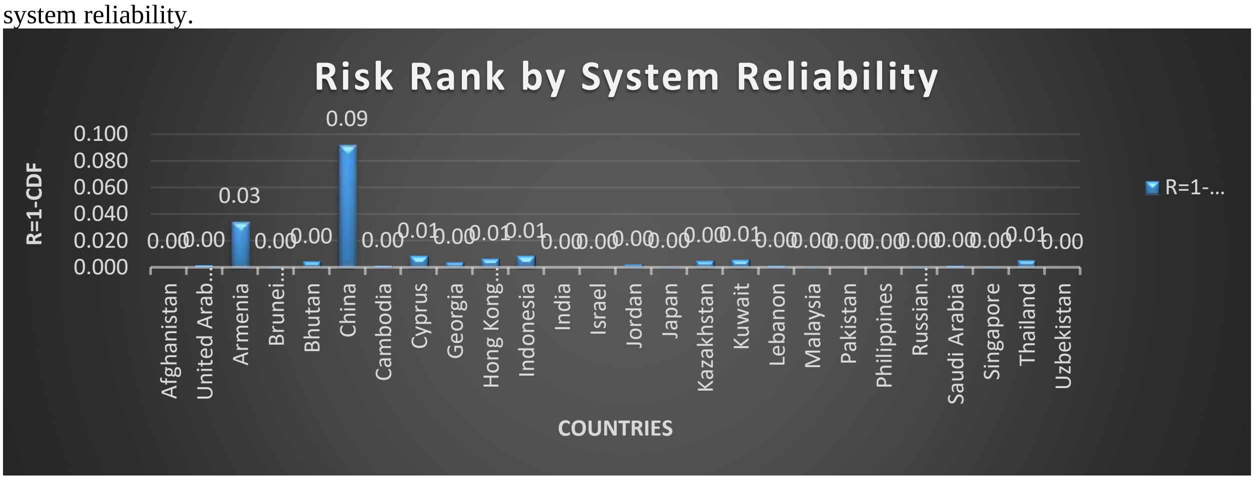

Figure 5: Risk Ranking by Weibull's System Reliability R

Seven countries have zero value of R. So these countries are dropped from risk ranking as per indicator of system reliability. These countries include Afghanistan, Brunei Darussalam, India, Israel, Pakistan, Philippines, and Uzbekistan. Zero value of R shows that the NPL rates in these countries are due to random chance, not due to the structural defect. The value of $R > 0$ shows that NPL functions as a system and that system is sufficiently reliable to produce NPL, and the failure comes from process or system.

3.1.2 Combined group analysis under Fisher-Tippett-Gnedenko Method

The second phase of the calculation is completed by using the mean of countries' twenty-eight operating quarters as an individual observation. Thus, there are 26 individual observed values for the group. As per literature, Weibull and Fréchet distributed data sets may be studied under the Generalized Extreme Value equation known as the Fisher-Tipett Gnedenko GEV equation (Embrechts *et al*, 1999).

As per Fréchet distribution equation, only values: $x>\mu$ are isolated for the QQ-plot determination. There are 11 values that meet this condition.

Table 9: Generating QQ-Plot for Linear Regression to Obtain Shape Parameter.

Country Name	Xobs	F(t)	/1-f(t)	Ln(A)	ln(ln(a)	ln(xobs)

Brunei Darussalam	1	0.061	1.065	0.063	-2.759	0.000
Kuwait	2	0.149	1.175	0.161	-1.823	0.693
Armenia	3	0.237	1.310	0.270	-1.308	1.099
Jnited Arab Emirates	4	0.325	1.481	0.392	-0.935	1.386
Bhutan	5	0.412	1.701	0.532	-0.632	1.609
Jordan	6	0.500	2.000	0.693	-0.367	1.792
Russian Federation	7	0.588	2.426	0.886	-0.121	1.946
Pakistan	8	0.675	3.081	1.125	0.118	2.079
Afghanistan	9	0.763	4.222	1.440	0.365	2.197
Kazakhstan	10	0.851	6.706	1.903	0.643	2.303
Cyprus	11	0.939	16.286	2.790	1.026	2.398

After finding the value of Xi and Yi, we determine the equation of linear regression represented by Y = a+ bX. In this case, the linear regression equation is Y =1.94+0.66X and shape of the Fréchet is α= 1/b = 1/0.66 = 1.52. TheThe positive value of shape parameter shows that that the trend is increasing concerning time. It means that the NPL trend in Asia is increasing with time. The countries' risk assessment of the industries is read with the scale of the failure. Maximum likelihood method is used for scale in the Fréchet distribution (Abbas and Yincai, 2012). The likelihood function is given by:

$$L_n(a,\beta) = a^n \beta^{na} \prod_{i=1}^{n} x_i^{-(a+1)} \; exp\left[-\sum_{i=1}^{n}\left(\frac{\beta}{x_i}\right)^a\right]$$

It is also suggested that by solving (∂ log Ln(αβ))/∂β=0), the maximum likelihood of the failure level may be found.

$$\hat{\beta}ML = \left(\frac{n}{t}\right)^{1/\alpha}$$

Where α is the shape of the curve determined by 1/slope; *n* = sample size which accounts for 11 countries meeting the condition $x > \mu$, and $t = \sum(1/X_i)^{\alpha}$.

In this case, the calculation shows that $\hat{\beta}ML$ =5.5. This value is the threshold of NPL level beyond which is considered high risk. Under the standard score method, seven countries were recognized as risky because 95% confidence interval was used.

Table 10: Identify Risk Industry under Generalized Extreme Value Method

Country Name	**Xobs**	**A**	**b**	**1.94+0.66X**	**Threshold**	**Risky**
runei Darussalam	1	1.940	0.660	2.6	5.5	No
Kuwait	2	1.94	0.66	3.26	5.5	No
Armenia	3	1.94	0.66	3.92	5.5	No
ited Arab Emirates	4	1.94	0.66	4.58	5.5	No
Bhutan	5	1.94	0.66	5.24	5.5	Yes
Jordan	6	1.94	0.66	5.9	5.5	Yes
ussian Federation	7	1.94	0.66	6.56	5.5	Yes
Pakistan	8	1.94	0.66	7.22	5.5	Yes
Afghanistan	9	1.94	0.66	7.88	5.5	Yes
Kazakhstan	10	1.94	0.66	8.54	5.5	Yes

Cyprus	11	1.94	0.66	9.2	5.5	Yes

The Countries may be ranked as per magnitude of risk resulting from their NPL ratios. Bhutan, Jordan, Russia, Pakistan, Afghanistan, Kazakhstan and Cyprus are considered extreme cases. These countries are the riskiest. The remaining four countries including Brunei Darussalam, Kuwait, Armenia, and the United Arab Emirates are considered within 0.95 confidence interval or tolerance level.

4 Conclusion

At the risk of oversimplification, there are two approaches to model the risk assessment of NPLs: classes of the time series models and macroeconomic models which are not as appropriate for the rare event. The study proposes that the NPLs are rare events. Consequently, this study proposes that EVT for risk modeling and assessment of NPLs as an alternative to two approaches in search of conclusive evidence. The data of 26 Asian countries from 2010 to 2016 for 28 numbers of quarters is used to test this alternative tool for risk assessment of NPLs. It is found that extreme value theory is comparatively and relatively appropriate tool for the risk assessment of non-performing loan. The EVT is more appropriate than the other approaches because it can be used to identify and rank the risk among the countries and industries. The results of the study out of twenty-six numbers of countries, NPL of seven countries exceeded than the threshold level determined under EVT. The Bhutan, Jordan, Russia, Pakistan, Afghanistan, Kazakhstan and Cyprus are considered extreme cases.

At the risk of generalization, the study has implications for the government policymakers and banking sector managers. The policymakers of the regulators and the other government institutions can use the EVT as an alternative tool to assess the risk of the NPLs in the policy formulation process. Similarly, the managers and executives of the banks can use this tool to ass the risk of NPLs. In this regard, they may develop, implement, and use the information systems that use the EVT before issuing the loans.

REFERENCES

Aurangzeb. (2012). CONTRIBUTIONS OF BANKING SECTOR IN ECONOMIC GROWTH : A Case of Pakistan. Economics and Finance Review, 2(6): 45–54.

Charras-Garrido, M. and Lezaud, P. (2013). Extreme value analysis: an introduction. Journal de la Société Française de Statistique, 154(2): 66-97.

Sontakke, R. N., and Tiwari, C. (2013). Trend Analysis of Non Performing Asset in Scheduled Commercial Banks in India. *International Journal of Application or Innovation in Engineering & Management (IJAIEM)*, *3*: 2319-4847.

Joseph, M. T., Edson, G., Manuere, F., Clifford, M., and Michael, K. (2012). Non performing loans in commercial banks: a case of CBZ Bank Limited in Zimbabwe. Interdisciplinary Journal of Contemporary Research in Business, 4(7): 467-488.

Cucinelli, D. (2015). The Impact of Non-performing Loans on Bank Lending Behavior: Evidence from the Italian Banking Sector. Eurasian Journal of Business and Economics, 8(16): 59–71. https://doi.org/10.17015/ejbe.2015.016.04

Farooq, U. S., S. A. Afridi, and W. Alam. (2010). "Banking Reforms in Pakistan-Impacts & Implications." *International Bulletin of Business Administration*, 7: 55-61.

Shah, A.S.S., Mushtaque, T., Jareeko, M., & Shaikh, F. M. (2014). Non-Performing loan and their Effect on the Economy, 3(12): 87–101.

Messai, A. S. (2013). Micro and Macro Determinants of Non-performing Loans, *3*(4): 852–860.

Makri, V., Tsagkanos, A., & Bellas, A. (2014). Determinants of non-performing loans: The case of Eurozone. Panoeconomicus, 61(2), 193-206.

Ghosh, A. (2015). Banking-industry specific and regional economic determinants of non-performing loans: Evidence from US states. Journal of Financial Stability, 20: 93-104.

Louangrath, P. (2015). Risk Assessment of Non-Performing Loans (NPL) Using Extreme Value Theory. The First International Comference on Multidisiplinary in Management. ICONIDA, Bangkok, Thailand., (February), 110–137. https://doi.org/10.13140/RG.2.1.2512.5840

Greenidge, K., & Grosvenor, T. (2010). FORECASTING NON-PERFORMING LOANS IN BARBADOS. *Journal of Business, Finance & Economics in Emerging Economies*, *5*(1).

Taleb, N. N. (2007). The black swan: The impact of the highly improbable (Vol. 2). Random house.

McNeil, A. J., Frey, R., & Embrechts, P. (2015). Quantitative risk management: Concepts, techniques and tools. Princeton university press.

Brooks, C. (2014). *Introductory Econometrics for Finance*. UK: Cambridge University Press.

Masood, O., Bellalah, M., Mansour, W., and Teulon, F. (2010). Non-Performing Loans and Credit Managers' Role: A Comparative Approach from Pakistan and Turkey. *International Journal of Business*, *15*(3), 347–362.

Bollerslev, T. (1986). Generalized autoregressive conditional heteroskedasticity. *Journal of econometrics*, *31*(3): 307-327.

Taylor, S.J. (1986) Modelling Financial Time Series, Chichester: John Wiley.
Taleb, N. N. (2007). The black swan: The impact of the highly improbable (Vol. 2). Random house.
McNeil, A. J., Frey, R., & Embrechts, P. (2015). Quantitative risk management: Concepts, techniques and tools. Princeton university press.
Embrechts, P., Resnick, S. I., & Samorodnitsky, G. (1999). Extreme value theory as a risk management tool. *North American Actuarial Journal*, *3*(2): 30-41.
Abbas, K., and Yincai, T. (2012). Comparison of Estimation Methods for Frechet Distribution with Known Shape, *1*(10), 58–64.
Embrechts, P., Resnick, S. I., & Samorodnitsky, G. (1999). Extreme value theory as a risk management tool. *North American Actuarial Journal*, *3*(2), 30-41.
Charras-Garrido, M. and Lezaud, P. (2013). Extreme value analysis: an introduction. Journal de la Société Française de Statistique, 154(2): 66-97.
Gilli, M., & kellezi, E. (2006). An Application of Extreme Value Theory for Measuring Financial Risk. Computational Economics, 27(2): 207-228.
Burnham, K. P., & Anderson, D. R. (2004). Multimodel inference: understanding AIC and BIC in model selection. Sociological methods & research, 33(2), 261-304.
Louangrath, P. (2015). Risk Assessment of Non-Performing Loans (NPL) Using Extreme Value Theory. *The First International Comference on Multidisiplinary in Management. ICONIDA, Bangkok, Thailand.*, (February), 110–137. https://doi.org/10.13140/RG.2.1.2512.5840
Masood, O., Bellalah, M., Mansour, W., & Teulon, F. (2010). Non-Performing Loans and Credit Managers' Role: A Comparative Approach from Pakistan and Turkey. *International Journal of Business*, *15*(3), 347–362.
Hill, B. M. (1975). A simple general approach to inference about the tail of a distribution. The annals of statistics, 3(5), 1163-1174.