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PORTFOLIO TILTING: HUNT FOR POSITIVE ALPHA THROUGH STYLE TILTS

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ABSTRACT. *A long discussion in literature exist to answer the question how a fund manager can generate extra returns? In order to answer the question this study is concerened with two aspects of this problem. First at discuss the portfolio construction process from separation theorem to modern style tilts. And in second step it provide imperical evidence for superior performance of style tilts. First of all active and passive style of management are compared. Data on returns is taken from KSE for five years and two sets of style based portfolios are constructed. Strong evidence is found in favor of active style of management. Actively managed funds are used as proxy for tilted portfolios. Data of Net asset value is taken from MUFAP. Tilted portfolios are tested for Size and value tilts. This study confirms higher performance of portfolio with style tilts.*
Keywords: Portfolio tilting, size, B/M value

Studies conducted by Basu [1983], keim [1983] and Fama & French [1992] have shown that stocks with smaller size "market capitalization" and high B/M value have generated higher returns for investors. Similar results can be observed for stocks selling at low multiples of their sales. The superior performance of value stocks and small cap stocks has provided new direction for portfolio tilting. This study is concerened with two aspect of this issue. First at discusses the portfolio construction process from separation theorem to modern style tilts. And in second step it provides imperical evidence for superior performance of style tilts.

Journey from separation theorem to style tilts It is very difficult to allocate proper assets mix when the investors have multiple options for investment. In earlier ages investors were limited to investment decisions that pertain to a specific security only. Concept of diversification with statistical measures by Markowitz [1952, 1959] opens a new era in portfolio theory. In contrast to single asset he introduce concept of large numbers. With large number in mind best available tilting option was to tilt the portfolio to maximum possible securities. Most prominent drawback of portfolio tilting based on large number is that it ignores risk factors associated with each security. Moreover, the underlying assumption of his work mean variance efficiency is of importance only if returns from securities are uncorrelated. Otherwise manager must tilt his portfolio to stocks with minimum correlation. Another problem arises when managers have to deal with multiple time period data. In order to address the issue researchers address the problem with different set of assumptions Fama [1970], Hakansson [1970, 1974] and Merton [1990]. These studies found that, portfolios that are constructed on the basis of multi period data are significantly different from single period portfolios. The difference arises because of the utility function "time series data".

Another important aspect of portfolio theory is the separation theorem. That is, if an investor has access to riskless asset, he will tilt his portfolio to mix of risky assets and risk free assets. The separation theorem thus proposed has three implications. First of all it provides ease in calculation. Problem faced by portfolio

manager has been solved by constructing a portfolio with combination of riskless assets and expected standard deviation spread. The two set of securities are joined by a tangent line from riskless asset. This tilting strategy can maximize the ratio of expected future returns with somewhat unknown probability minus the return on the difference of riskless assets and return on assets with defined standard deviation.

Another important implication is the mutual fund theorem. It this particular time period, an important question was raised by Rose [1978]. The basic assumption of constant lending and borrowing risk free rate was very crucial. It is not always available to each and every individual. If we relax this assumption the tilting decision toward mutual fund theorem lose it significance up to 30% [Fama & French, 2004]. For now let us discuss with a simple example. Keeping our discussion limited to Markowitz efficient frontier, if there is different lending and borrowing rate for riskless assets then four mutual funds must be created. Thus investor will create his portfolio by tilting his decision toward two funds of risk free assets and two funds of risky assets that lies on efficient frontier. Third implication of separation theorem is it helps in explaining the estimation of the inputs that needs to be included in a portfolio.

Another point of concern is the calculation of correlation matrix. Moreover estimation of efficient portfolios through quadratic equation programming and difficulty of educating portfolio managers to calculate risk and return tradeoffs as well as the relation of covariance matrix to returns and standard deviation makes the theory more complex.

Most important of these areas is the first one that is the computational efficiency. That is to provide such inputs that can maximize the overall returns of portfolio. If portfolio manager solve this problem they will be able to overcome the next two problems in the long run Elton [1976]. In order to solve the problem of efficient inputs calculation of covariance matrix was required. The principal technique develop to solve the covariance problem was index models. Single and multiple index models were developed with the passage of time. The first and simplest single index model was first discussed by Markowitz [1952] and was developed by Sharpe [1967]. This model was based on single index that is why called market index.

$$Rit = \alpha i + \beta i Rmt + eit \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (i)$$

Where R_{it} is the return of the specific stock or security "i" which is trade in time period "t", "ai" is the unique expected return of security "i", Beta of stock "i" is the sensitivity of stock "i" to the overall market movements, which is discussed above. R_{mt} the log return on the market as a whole within a specific time period t, and e_{it} is the error term which is use to reflect the unique risky return of security "i" in period "t". This unique return is assumed to have mean of zero and variance r^2_{ei}. Index models decrease the number of inputs required to estimate a portfolio performance. Beside ease in performance calculation it gives more realistic results with both single and multiple time period data. They also increase accuracy in calculating covariance for large set of data Gruber [1973]. Index models provide good results with both positivist and interpretive approach of fund management Elton [1997]. Index models also give freedom in comparison of a specific stock to overall market or any sector.

With the passage of time researcher started their struggle to explore the reality with more power full models. These models are known as multi index models. The trial model that was formulated was

$$R_{it} = \alpha_i + \sum_{i=1}^{n} \beta_{ij} I_{jt} + e_{it} \qquad i = 1.2.3 \ldots\ldots\ldots, n \ldots\ldots\ldots\ldots (ii)$$

In the given equation β_{ij} is the sensitivity of security I to index j. I_j means the j_{th} index and J means total number of indexes employed. Multi index model got significant importance with the passage of time. But main problem with multi index model is to choose index. In early days of multi index models many statistical measures were used to chose index. Some scholars used factor analysis while some used variance-covariance matrix of returns Roll & Rose [1980], Brown & Weinstein [1993]. Another approach used to calculate multi index model was pre specification of a specific structure. Three different approaches can be used to select an index. These are market plus industry indexes that is discussed by Cohen & pogue [1967]. Another approach use surprises in basic economic indexes Chen [1986]. Surprises in economic indexes can be reflected through production or inflation in the economy. Third approach used portfolio of traded securities. Traded securities portfolio is getting more importance in recent days. Through this approach indexes created are composed of an index of small minus large securities Fama & French [1992]. In today's modern world portfolio of traded securities is gaining paramount importance both in academia and in policy circle. Factor analysis is only good to the point of confirming a specific factor. It is the portfolio of traded securities that is able to explain returns on any security up to 95%. Some basic reasons for multi index models popularity are that they can be used to

provide inputs to portfolio. Multi index models are building blocks of arbitrage pricing theory. At the same time they can be used to understand the sensitivity of any stock to multi index simultaneously. And last but not least is it can be used to evaluate mutual fund performance. Moreover the flexibility of these models increases their worth. Portfolio manager can use them to understand fund sensitivity to economic indicators. The same manger can use these models to reformulate his portfolio by tilting his investment decision to attractive stocks like small stocks or HML Fama & French [2004].

So far discussion is base on portfolio selection process. If we assume that CAPM holds and we ignore the observation of Fama [1968] that the residuals are correlated and that we allow short sales, then the solution presented by Elton [1976] will be reduces to Treynor & black [1973]. They said that, the investment in any stock must be equal to proportion of alpha and the variance of residual risk. So in case of multi index models returns are generated by multi dimensional aspects. With change in factors the residual risk is changed, so manager has to include another risk measure for new dimension. Similarly, if some securities are mispriced another index has to be introduced in order to acknowledge the mispriced security. The only case where manager face problems in reflecting multi index models is any macroeconomic fluctuation. Macro economic factors are difficult to cope with because they are not symmetrically distributed Elton & Gruber [1992]. Keeping in view these complexities a manager has to construct a portfolio which is the combination of different portfolios. Portfolio must be able to grasp the overall effect of macroeconomic factors like the market index. Moreover, it should be combined with other indexes that can grasp the fluctuation in each security Elton & Gruber [1992]. Latter on Fama & French [1992, 2004] developed their three factor model to address the same issue.

Performance evaluation of tilted portfolio Study shows that portfolio management is all about value addition to overall wealth of investors. A passive strategy can minimize the risk associated with any investment decision but it is baseless to hope for higher returns or having a vision to outperform the overall market. In order to achieve incremental value addition to overall portfolio returns it is important to have an active strategy. Active strategy involves tilting decisions at regular basis. It is very important for a manager to evaluate outcomes of his tilting decisions. Cowles [1993] has compared performance of managed funds with broader index and found that managed fund underperformed index funds in long term. Major shortcoming of his work is that he does not include risk factors in his technique. Early studies that got significant importance were conducted by Sharpe [1966], Treynor [1965], and Jensen [1968, 1969]. These studies evaluated portfolio performance using risk factors. Some of those studies use total risk "standard deviation" of a security Sharpe [1966], Friend [1970], while some focus on only systemic risk "Beta" Treynor [1965], Jensen [1968]. Each of these studies concluded that an effective portfolio is one which is tilted toward combination of two types of assets that are risky and risk free assets. Keeping our discussion limited to Jenson alpha. the intercept from time series regression of returns of individual security, market index and risk free rate. The general equation used in Jensen model was

$$R_{pt} - R_{ft} = \alpha_p + \beta_p(R_{mt} - R_{ft}) + \epsilon_{pt} \dots\dots\dots\dots (iii)$$

Jensen alpha can be used as proxy for calculating tilt in portfolio. Importance of selecting an appropriate index for alpha calculation was discussed by Roll [1978]. Jensen alpha has significance in portfolio evaluation but we cannot rely only on a single index. In past few decades small stocks have outperformed the market. So, we cannot restrict our self to portfolio that is the combinations of index funds and riskless assets only.

Moreover, such models are needed that can explain the nature of returns, that a tilted portfolio generates in long period of time. Performance of tilted portfolio to different assets can be attributed to model with "N" number of indexes Grinblatt & Titman [1987]. Same arguments were presented by APT and roll in his famous criticism on CAPM Ross [1978].

Ippolito [1989] discussed different issue. He says that tilting portfolio to high load or low load funds can increase performance. His sample was composed of large number of small stocks. Keeping in view the importance of small stocks most of the fund managers tilt their portfolio to small stocks. Thus, single index models were updated to a new index that can grasp the importance of small stocks. Jensen model was updated for small stocks as

$$R_{pt} - R_{ft} = \alpha_p + \beta_p(R_{mt} - R_{ft}) + B_{ssp}(R_{sst} - R_{ft}) + \epsilon_{pt} \dots\dots\dots (iv)$$

In above model R_{sst} is return on small stocks in time t rest of the variables are same as discussed above. Researcher used this model to measure alpha for portfolios. It was interesting to note that when the same model was implemented on the data used by Ippolito [1989] the results observed were reversed. Almost all of the funds underperformed the market. Even the portfolio with small stocks underperformed the passively managed funds. It also shows that load funds underperformed no load funds. Such findings bring more confusion in multi index model. More studies by Lehmann & Modest [1987], Connor & Korajzcyk [1991] were conducted to evaluate portfolio performance with statistically derived models.

Another approach that is used to developed multi index model is the type of securities held by managers. The pioneering work in this approach can be traced to Sharpe [1992], Elton [1996] and Blake [1993]. Elton model is based on four indexes i.e. S&P 500 index, size based index, bond index and growth value index. The last factor is used to grasp the effect of non specialized stocks in overall portfolio. In contrast to Elton, Sharpe uses domestic and foreign bonds and stocks to create a 12 index model. Model developed by Sharpe and Elton provides better returns then those used before, but the issue of appropriate model is still not solved.

Another important aspect of portfolio performance is evaluated through return ratio. Instead of using Sharpe ratio "difference between average return of overall portfolio with risk free assets and then dividing it by overall risk of portfolio that is standard deviation of the portfolio" Sharpe developed another ratio that is known as generalized Sharpe ratio which uses time series data of historical returns. Generalized Sharpe ratio for multi index portfolio through time series analysis cab be written as

$$\text{Sharpe Ratio} = \alpha_p / S_p \text{.....(v)}$$

If we talk about superior performance manager must adopt both active and passive strategies to generate positive alpha. As discussed above in start of portfolio theory the asset pricing model of Sharpe [1964], Lintner [1965] and black [1972] has long shaped the decision pattern of fund mangers based on risk and return preferences. Combination of both these strategies give rise to tilted portfolios. Keeping in mind the criticism on beta as the sole measure of cross section of returns, Size effect Banz [1981] got significant importance. He explains that size of the security that is its ME [market price times shares outstanding] contribute significantly to the explainatory power of Beta. Similarly another prominent contridiction is positive relation between leverage and average retuns Bhandari [1988]. Another important factor that is highlighted in the literature in last two decade is the ratio of a firm's book value of common equity BE, to market value of that share ME Stattman [1980]. Positive effect of B/M is also explained in studies conducted by Rosenburg, Reid,& Lanstein [1985]. Another important factor which can be used by fund manger to tilt his portfolio is E/P high lighted by Basu [1983]. It is stated that securities with higher earnings and small prices are more suitable for style tilts Ball [1987]. It is expected that Balls [1987] proxy can grasp the impact of all style factors, such that, it can accomudata size factor, B/M and leverage.

Reaserch shows that Beta was powerfull measure of risk in period before 1970's Reinganum [1981]. So it would be better for a manger to rely on multivariate relation i.e. beta, size, leverage and book to market ratio rather then focusing on only univariate relationship with beta. More recent studies have been conducted to highlight the effect of size and book to market value on stock returns. Some of these studies are summarised here.

Size and B/M reflects the behavior of earnings, the market factor and size factor reflects the earning pattern but there is not sufficient support for B/E Fama & French [2012]. Size has strong relationship with earnings of a security while HML can be doubted in some cases Brailsford [2012]. Another study that negates the behavior of Fama & French [1992] three factors is conducted by Knez & Ready [2012] they said that risk premium on size totally disappears with extreme observations.

3.Rationale behind tilting decision: The capital asset pricing model is used in finance to determine an approporiate rate of return on any investment decision specifically return on stocks. CAPM has its application in the field of portfolio theory as it take in to consideration a major risk factor that is maket risk. But recent criticism on beta makes it difficult for fund mangers to base their decision just on beta. The reason behing this confusion is the explainatory power of Beta. Fama & French conducted test on mumber of portfolios and found that Beta is able to grasp the deviation upto 70% only. There are other factors that add to the explenatory power of CAPM significanly. They stressed on two more important factors i.e. size of the stock in a portfolio and the value factor that is price to book value of the stock within a portfolio.

Style Tilt: Tilting toward Small Stocks : Style tilts to portfolio increases the chances to beat the market. Size factor shows a unique risk associated with any security. Usually small securities are not secured that's why they are riskear and show irregulare return in long period of time as compared to large stocks that are comparitively more stable. In long run small stocks have generated hihger returns then high market capitalization Fama & French [1992, 2004, 2012]. They studied the behavior of two style tilts i.e. size and book to market values related with earning behavior. This model has been tested using data from 1980 and found that three factor model better explain the returns rather then other factor based models Halliwell, Heaney & Sawicki [1999].

4.Style Tilts: Tilting toward Value Stocks: Another important style tilt is in the shape of high book to market value. Tilting portfolio to value stocks also helps mangers to increase their portfolio returns. Value stocks are those companies that usually have lower earnings growth rates, they pay high dividends and they have higher book value as compared to their prices. Fama & French also studied the nature of gworth stocks and value stocks and found that these two groups of stocks behave differntly in long run. Combination of size and market factor B/M results in better performance as shown in different studies conducted by Durack, Durand & Maller [2004], Gaunt [2004], Gharghori, Chan & Faff [2006, 2007] and Faff [2001, 2004].
However conclusions from some studies showed that B/M stocks in some cases have weak performance, although studies by Gharghori, Chan & Faff [2006, 2007] indicate that increasing the weights of HML stocks would lead to premium that is positive and significant.

5.Empirical Evidence for better performance of style tilts: The basic objective of this study is to shed light on the rational behind portfolio tilting. In order to test the tilting decision and effectiveness of fund mangers, this study is using Factor based models FBM. As we discussed that adding a style tilt in the form of size and value /growth stock increase the excess returns, so this study will test the impact of size tilt and value/growth tilt on excess returns with the help of Fama & French three factor model. This study will follow the following model.

$$R_{it} - R_{ft} = \propto_i + B_i(R_{mt} - R_{ft}) + S_i SMB_t + H_i HML_t + \epsilon_{it} \ldots\ldots\ldots\ldots (vi)$$

Number of sources have been used to collect data. Data on open end mutual funds have been dowmloaded from MUFAP while data on maket index has been downloaded from KSE. In order to provide support for style tilts this study generated two sets of style oriented portfolios. First set of portfolios are based on size effect and the second are based on B/M. These portfolios are constructed on two basic assumptions.
Each portfolio is constructed on the basis of single style factor with equal weight to each stock in the portfolio. Portfolios constructed on single style factor are managed passively for five year period. The manger will remain passive throughout the process/period; He does not change the composition of the portfolio. Data on individual stocks are obtained from Yahoo finance and ZHV official sites. Size based sorting is carried out to create different size oriented deciles. Each decile is converted to individual portfolio thus ten size based portfolios are created. Due to high correlation between size and beta this study converted the size based portfolios to size-beta based portfolios as discussed by Fama & French [1992]. Same procedure is repeated for ten B/M based portfolios.
In order to calculate the return of each size-beta and B/M based portfolio the following equation is used.

$$R_p(Size - beta) = (W_1 * R_1) + (W_2 * R_2) + (W_3 * R_3) \ldots\ldots (W_n * R_n) \ldots\ldots (vii)$$

Where "R" is the respective return and "W" is the weight of each security in the portfolio. Similarly for set of 10 risky assets that makes a single portfolio it risk can be calculated with the extended form of this formula as

$$\delta^2(r_p) = W_a^2\delta^2(r_a) + W_b^2\delta^2(r_b) + W_c^2\delta^2(r_c) + 2W_aW_b cov(r_a r_,) + 2W_aW_c cov(r_a r_c) + 2W_nW_n cov(r_n r_n) \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (viii)$$

Table No 4.1:

Size based portfolios average monthly return for period 2008 to 2012

S.NO	AR 07/08	AR 08/09	AR 09/10	AR 10/11	AR 11/12	Total Return	Average
Size Portfolio 1st Deciles	0.17	0.246	0.174	1.51	1.98	0.816	
Size Portfolio 2nd Deciles	0.19	0.268	0.158	1.92	1.90	0.8872	
Size Portfolio 3rd Deciles	0.10	0.141	0.160	0.19	0.68	0.5707	
Size Portfolio 4th Deciles	0.09	0.240	-0.150	0.18	0.98	0.2611	
Size Portfolio 5th Deciles	0.004	0.222	-0.145	-0.15	0.46	0.1731	
Size Portfolio 6th Deciles	0.05	0.191	-01.37	0.01	0.35	-0.0378	
Size Portfolio 7th Deciles	0.007	0.178	-0.121	0.00	0.08	-0.0625	
Size Portfolio 8th Deciles	0.003	0.160	-0.131	-0.01	-0.25	-0.0084	
Size Portfolio 9th Deciles	-0.02	-0.131	-0.024	-0.06	-0.05	-0.0513	
Size Portfolio 10th Deciles	-0.00	-0.001	-0.009	-0.04	-0.07	-0.0405	

With increase in size of the portfolio the returns decreases dramatically. Thus, confirming the results of Fama & French [1992]. Second type of style tilt that got significant importance is tilting toward value stocks. That is tilting portfolio toward those stocks that has high Book to Market value. Second set of portfolios are created following Fama and French cited above. Average returns of B/M portfolios are presented in table below

Table No 2:

Portfolios based on Book to Market Value for the period 2008 to 2012

S.NO	AR 07/08	AR 08/09	AR 09/10	AR 10/11	AR 11/12	Total Return	Average
B/M Portfolio 1st Deciles	-0.01	-1.18	-0.70	-0.90	-0.450	-0.648	
B/M Portfolio 2nd Deciles	-0.01	0.019	-0.12	-0.14	-0.554	-0.161	
B/M Portfolio 3rd Deciles	0.10	0.252	-2.51	-0.54	-0.591	-0.657	
B/M Portfolio 4th Deciles	0.09	-0.67	-1.25	-0.24	0.390	-0.336	
B/M Portfolio 5th Deciles	0.41	0.876	0.012	-0.14	0.740	0.3796	
B/M Portfolio 6th Deciles	0.52	0.732	0.862	0.257	1.206	0.7154	
B/M Portfolio 7th Deciles	0.73	1.178	0.125	1.463	1.156	0.9304	
B/M Portfolio 8th Deciles	0.89	1.189	1.456	2.412	2.497	1.6888	
B/M Portfolio 9th Deciles	-0.02	1.698	1.987	2.568	6.384	2.5234	
B/M Portfolio 10th Deciles	-0.00	2.254	1.815	3.457	8.445	3.1942	

Noticeable change in returns can be observed with increase in B/M value in all the columns. The increase in return down the column shows that value stocks are giving high returns as compare to growth stocks. Growth stocks shows average return for five year period -0.648, -0.161 and -0.657. Value stock shows average return for five years as 1.6888, 2.5234 and 3.1942. It confirms that value stocks perform different from growth stocks. As value stocks are more risky that's why they are giving higher return then growth stocks.

6. Can style tilts add to return of the portfolio: Testing for style tilting: Tilted portfolios are not focusing on index tracking rather it combines some portion of broader index with risk free rate "following the basics of separation theorem" and then adding style tilt to the mix of securities. Tilted portfolio is one which generates extra returns after adjusted for different level of risk associated with the fund. This study is using the following technique to grasp the effect of tilting on a portfolio.

Let us suppose an investor whose portfolio is composed of riskless assets and few risky assets. Let another supposition be that his portfolio is tracking the broader index i.e. Karachi Stock Exchange. His portfolio is composed of N risky assets i=1,2,….. n. Let "π_p" be the portfolio and "W_i" be the weights of each security in overall portfolio which adds up to 1. Suppose the investors want to tilt his portfolio from broader index portfolio "π_p" to some more risky assets say "q". His tilt in this situation can be define as

$$\pi_{p\rightarrow}\pi_p+\epsilon\left(\pi_q-\pi_p\right) \ldots\ldots\ldots\ldots\ldots\ldots\ldots\text{(ix)}$$

"ϵ", is loading parameter of new portfolio. The combination of risky and risk free assets is now transformed to a new portfolio "π_q". This portfolio is formed by liquidating some portion of "π_p" and used the balance to invest in some other asset to tilt the portfolio "π_p" to "π_q". In order to assess the effect of this tilt we have to assume that the new investment decision has finite variance and the investors value all his decision purely on mean variance efficiency that is maximum return "μ" with minimum variation "δ^2" Markowitz [1952, 1959]. Sharpe theory can be used to explain the nature of investment for those investors who has mean variance preferences.

$$S=\frac{E(R)}{\sqrt{v(R)}}\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\text{(x)}$$

If the tilt toward "q" has incremental benefits it will increase Sharpe ratio significantly. Similarly tilting effects has strong connection with Jensons alpha. If we have two set of securities, one is benchmark "p" and the other one is any style stock say "q", in this case Jenson alpha is the intercept parameter which is observed through linear regression equation

$$E(R_q)=\alpha+\beta E(R_p) \ldots\ldots\ldots\ldots\ldots\ldots(xi)$$

Where R_p the excess return on portfolio "p" and β is is the slope of regression equation. In this particular case Beta is measure by

$$\beta=\frac{Cov(R_p,R_q)}{V(R_p)} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(xii)$$

If market or the benchmark is mean variant efficient then Jensen Alpha α equals to 0, if we liquidate some portion of our portfolio "p" and use its liquidated value to purchase asset "q", it will increase the overall value of portfolio "q". It means that the tilting loading "ϵ" will give positive incremental shift to the portfolio. If this assumption holds and the incremental tilt gives positive benefits then for new portfolio "q" Jensen alpha will be greater than zero. In this specific scenario we can use the generalized Sharpe ration to gauge the tilting effect. This ration is combining Jensen Alpha with the standard deviation of the portfolio

$$S(0)=\frac{\alpha}{\delta_p} \ldots\ldots\ldots\ldots\ldots\ldots\text{(xiii)}$$

Generalized Sharpe ratio shows the effect of incremental benefits of tilting but it does not shows the magnitude of tilting. Moreover, it lacks the ability to show that which style factor is playing key role in incremental returns. That's why this study is using Fama & French three factor model in addition to generalized Sharpe ratio to test the performance of actively managed mutual funds.

Factor based model will also test the hypothesis for significance of style tilts. Through factor based model this study will show that which specific style tilt can increase the performance of mutual fund manager. The model used is as follow

$$R_{it}-R_{ft}=\propto_i+B_i(R_{mt}-R_{ft})+S_iSMB_t+H_iHML_t+\epsilon_{it} \ldots\ldots\ldots\ldots(xiv)$$

Where: $(R_{mt}-R_{ft})$ represents the tilt toward broader market index. SMB_t Shows that whether a particular fund is tilted toward small stocks or not? HML_t Shows that whether a fund is tilted toward growth stocks or value stocks?

For the time assume that the fund manager is not tilting his portfolio. he has designed his portfolio based on a single style factor and managed his portfolio passively for five years period. The performance of such a portfolio is tested with the following factor based model.

$$R_{it}-R_{ft}=\propto_i+B_i(R_{mt}-R_{ft})+S_iSMB_t+H_iHML_t+\epsilon_{it} \ldots\ldots\ldots\ldots(xv)$$

If any of these portfolio generates positive alpha it will show that the passive strategy with style tilt can generate higher return as compare to market. If none of them or maximum of these portfolio generate negative alphas it will show that despite the higher risk associated with style tilt these funds cannot outperform the

market. Regression analysis for all these twenty portfolios have been performed and found that despite the fact that each security in the portfolio adds his unique risk neither of the portfolios is able to outperform the market with positive alpha. Their poor performance indicates that these portfolios must be tilted toward more indexes. This study used open end mutual funds as proxy for tilted portfolio and tested performance of 140 funds with factor based models.

Table No 3:

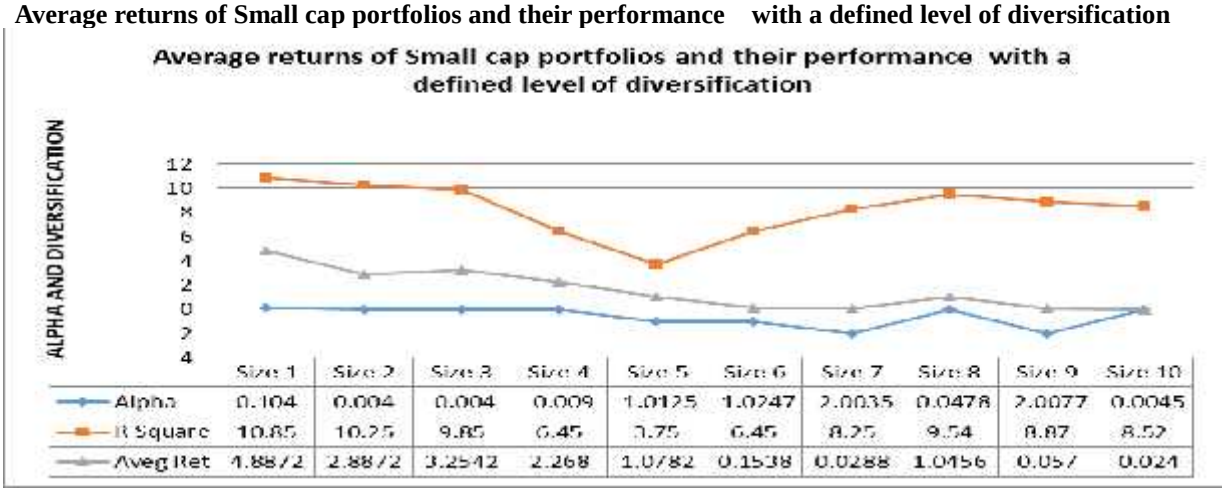
Passively managed funds based on Size and Book to Market

S #	Portfolio	Alpha (Managers performance)	R Square
1	Size 1	0.104	10.85
2	Size 2	0.004	10.25
3	Size 3	-0.004	09.85
4	Size 4	-0.009	6.45
5	Size 5	-1.0125	3.75
6	Size 6	-1.0247	6.45
7	Size 7	-2.0035	8.25
8	Size 8	-0.0478	9.54
9	Size 9	-2.0077	8.87
10	Size 10	-0.0045	8.52
11	B/M 1	-1.452	6.54
12	B/M 2	-4.585	7.34
13	B/M 3	-5.551	7.85
14	B/M 4	-3.258	9.53
15	B/M 5	-2.745	4.75
16	B/M 6	-3.752	8.88
17	B/M 7	-2.582	3.46
18	B/M 8	-1.245	7.97
19	B/M 9	0.0004	9.15
20	B/M 10	0.0058	6.57

After adjusting for three risk factors i.e. Market risk, Size and B/M we can notice that neither of the fund generate positive alpha. This shows that almost all the funds are underperforming the market in the five year time period. Except the two high risky “Size” based portfolio and two high “B/M” portfolios. But it’s not practical to hold such portfolios in practical life. A mutual fund is more restricted to do so because of the following reasons.

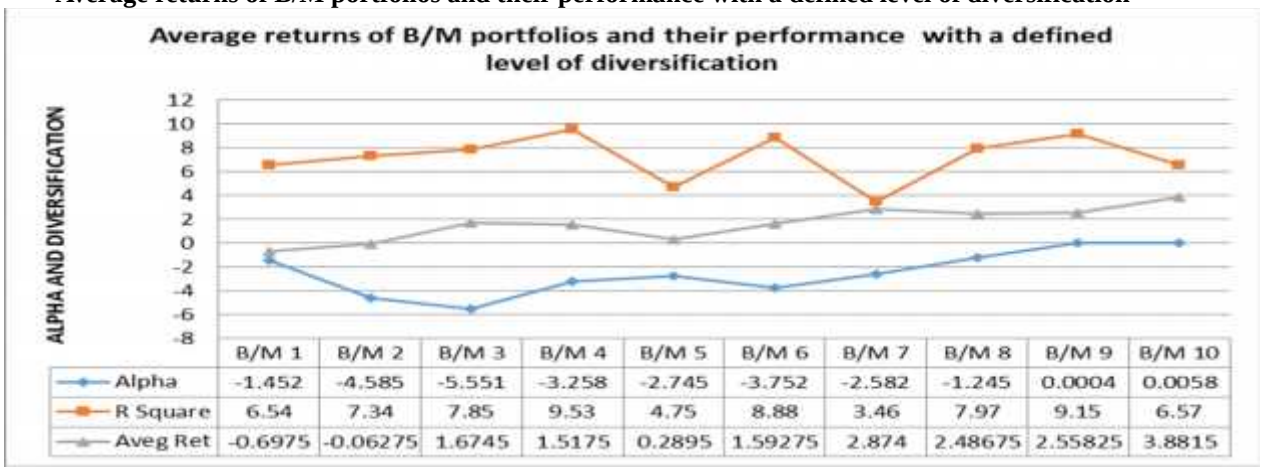
A fund manger cannot invest all his resources in a single company. In order to protect investors Security Exchange Commission of Pakistan compels fund managers to invest their resources in minimum ten types of stocks. Due to high pressure from investors a fund managers cannot take high risk. Moreover, manger cannot wait for 4 or 5 years period of time because he has to accommodate the operational cost of fund management and he is liable to pay periodic payments to unit holders on regular basis. That is why a manger has to remain active must tilt his portfolio on regular basis to combination of risky assets such as Small stocks and Value stocks and risk free assets such as treasury bills, gold and cash etc. such type of style tilt will provide him better results and the risk is also minimized because of the diversity in mutual fund.

Figure 1:
Average returns of Small cap portfolios and their performance with a defined level of diversification



	Size 1	Size 2	Size 3	Size 4	Size 5	Size 6	Size 7	Size 8	Size 9	Size 10
Alpha	0.104	0.004	0.004	0.009	1.0125	1.0247	2.0035	0.0478	2.0077	0.0045
R Square	10.65	10.25	9.85	6.45	5.75	6.45	8.25	9.54	8.87	8.52
Aveg Ret	4.8872	2.8872	3.2542	2.268	1.0782	0.1538	0.0288	1.0456	0.057	0.024

Figure 2:
Average returns of B/M portfolios and their performance with a defined level of diversification



	B/M 1	B/M 2	B/M 3	B/M 4	B/M 5	B/M 6	B/M 7	B/M 8	B/M 9	B/M 10
Alpha	-1.452	-4.585	-5.551	-3.258	-2.745	-3.752	-2.582	-1.245	0.0004	0.0058
R Square	6.54	7.34	7.85	9.53	4.75	8.88	3.46	7.97	9.15	6.57
Aveg Ret	-0.6975	-0.06275	1.6745	1.5175	0.2895	1.59275	2.874	2.48675	2.55825	3.8815

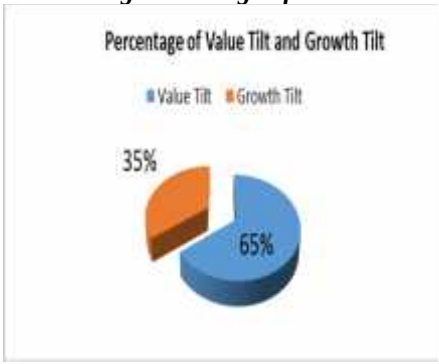
7. Testing for actively managed diversified funds with style tilts Fund managers have to create well diversified portfolio beyond separation theorem. In order to provide support for style tilting this study tests actively managed portfolios as proxies for tilted portfolios. Moreover, preliminary interviews with mutual fund managers confirm that they are managing their funds on daily basis with style tilts. The factor based regression is repeated for all those funds which have data available for minimum five years. Before using the estimation technique normality of the data has been checked through Skewness and Kurtosis. This study use White test to check Heteroscedasticity following the methodology of Asteriou [2006]. Durbin Watson statistics is used to test autocorrelation in the data. After clearing the data the analysis were performed and results are displayed in appendix.

43 of the open end mutual funds have higher performance in the form of positive alpha, showing that 61.11% of the total target population is outperforming the market. It means that after adjusting for three types of risks i.e. market risk, size risk and growth to value factor these fund still have positive alpha. Most of the times style tilt is significant for each fund outperforming the overall broader market index. R square value for open end funds shows that most of the funds are well diversified. Their R square value is above 50. It indicates that these funds are tilted toward different type of investment options and are well diversified.

Figure No 3:
Pie Chart showing performance of Open End Mutual Fund

Figure No 4:
Pie Chart showing Percentage of Small stocks Tilts and Large Stock Tilts

Figure No 5:
Pie Chart showing Percentage of Value Tilt and Growth Tilt



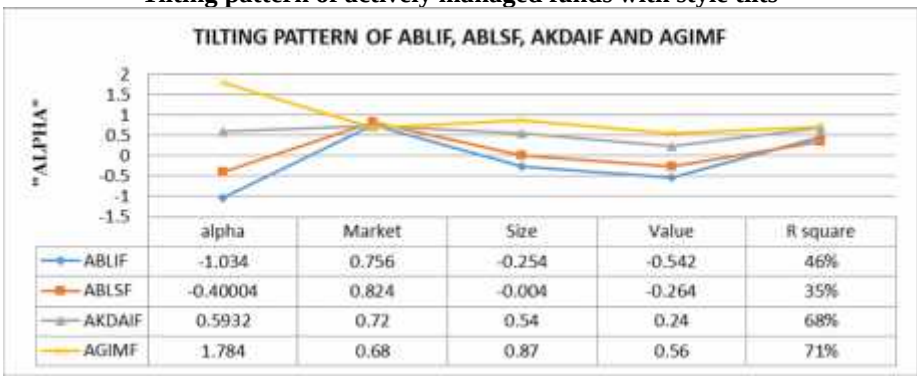
8. H1: If a manager adds style tilt in the shape of Size to his portfolio, this tilt has positive and significant effect on excess returns. 65% of the funds are tilted toward small stocks, 35% of the funds are tilted toward large capitalization stocks. Higher positive value "greater than 50" indicates that these funds are more tilted toward small stocks. P value for the size tilt is less than 0.05 for all those funds that have positive alpha. It provides sufficient evidence that we can accept H1. So we can generalized that adding style tilt in the shape of small size add to the performance of fund managers. Our results are similar to number of studies in literature. Stocks that are smaller in size are able to generate superior performance Basu [1983]. Same results were concluded by Keim [1983]. Similarly our results are similar to Fama & French [1992, 1998] and Brown [2008].

9. H2: If a manger adds style tilt in the shape of Value stocks to his portfolio, this tilt has positive and significant effect on excess returns. 65% of the funds have value tilt. Analyzing the table we can see that 47 of 72 funds have positive coefficient value for Value tilt. The higher the value of the coefficient the higher the fund is tilted toward value stocks. All the funds with value tilt has positive alpha, which confirms their superior performance as compare to market. 25 of the funds have negative value of coefficient of value tilt. It indicates that these funds are tilted toward growth stocks. P value for funds that are tilted toward value stocks are less than 0.05 which provide enough support for the acceptance of hypothesis. So we accept the hypothesis that adding style tilt in the form of value stocks can add the performance of fund. Our results are similar to those of Campbell & Vuolteenaho [2004], Fama & French [1992] and Fama & French [2004].

10. H3: portfolios with style tilt generate positive alphas as compared to randomly generated passively managed funds. Those mangers who have added style tilt to their fund will generate higher performance in for m of positive alpha. 61% of the fund outperforms the market with positive alpha. 100% of these funds with superior performance have style tilt. We can observe that superior performing funds have both value tilt and size tilt and are significant. P value for style tilts of all those funds which has positive alpha is significant "<0.05". Thus it provides enough support for the fact that adding style tilt can help the manger to increase his performance.

11. Tilting pattern of actively managed funds with style tilts Results of Fama & French three factor model and generalized Sharpe ratio confirms that funds with style tilts have higher probability to outperform the market. Most of the funds with negative alpha are just mimicking the market. It can be witness in the graphical analysis below. Three sets of funds are compared in the graphical analysis below. We can see that ABLIF and ABLSF in fig 6 start up with a negative alpha [-1.03 and -0.40]. Both these funds have positive value for tilt toward market index almost 75 plus. But when it comes to style tilts the fund managers hesitates to add it. Their funds are tilted toward large size stocks. In contrast to these two funds AKDAIF and AGIMF in fig 7 are mimicking the market index but their managers seem to be risk lovers. When it comes to style tilts these funds shows value far above 0.5. It means that both these funds are tilted toward small stock funds. Same pattern can be observed for AKDOD and AGIF. Both are able to beat the market because they are adding more risk to their portfolio in the shape of size and value tilts.

Figure 6:
Tilting pattern of actively managed funds with style tilts



	alpha	Market	Size	Value	R square
ABLIF	-1.034	0.756	-0.254	-0.542	46%
ABLSF	-0.40004	0.824	-0.004	-0.264	35%
AKDAIF	0.5932	0.72	0.54	0.24	68%
AGIMF	1.784	0.68	0.87	0.56	71%

Figure 7:
Tilting pattern of actively managed funds with style tilts

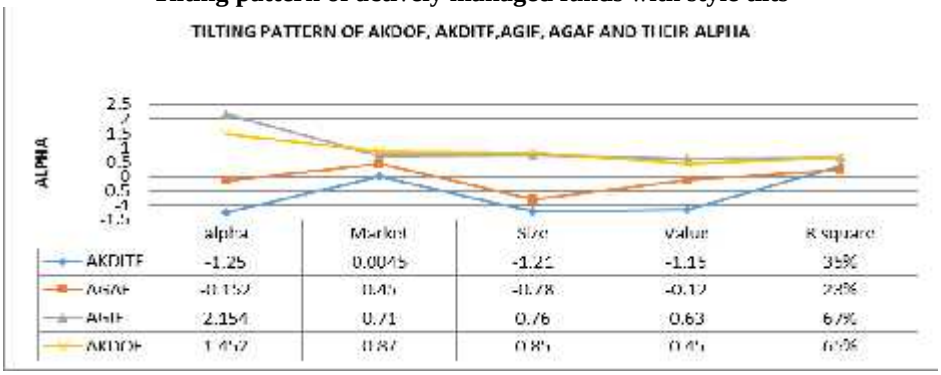
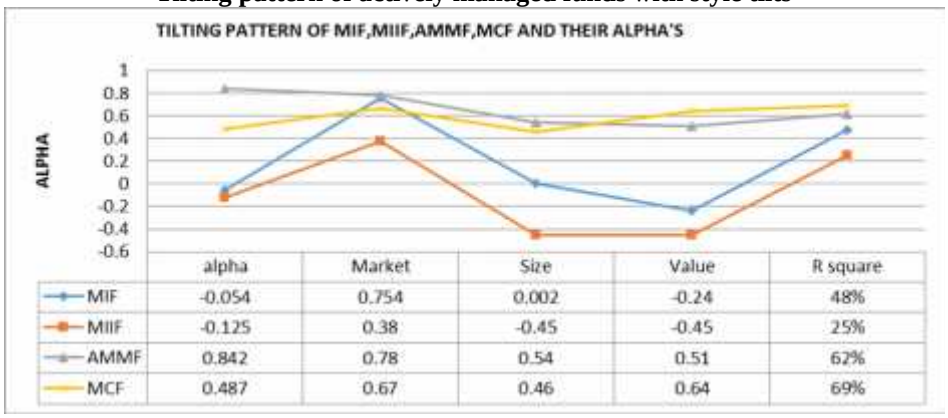


Figure 8:
Tilting pattern of actively managed funds with style tilts



12. Conclusion A long discussion in literature exist to answer the question how a fund manager can generate extra returns? In order to answer the question this study is concerened with two aspect of this problem. First at discuss the portfolio construction process from separation theorem to modern style tilts. And in second step it provide imperical evidence for superior performance of style tilts. First of all active and passive style of management are compared. For this purpose two sets of style based portfolios are constructed. Each portfolio is composed of ten stocks. One set is composed of ten portfolios based on Size tilt and second set of portfolios are composed of B/M tilt. Strong evidence is found in favor of active style of management. Once it is confirmed that active portfolio management is better than style tilts are focused. Two types of style tilts are tested here in this study. Actively managed funds are tested for Size and value tilts. Most of the funds with style tilts show positive alphas. Future studies must be conducted by relaxing the assumptions of this study. Portfolios with different weight should be created. They must not be held passively rather active management strategy should be adopted before testing their performance.

Appendix:

Table 6: Performance of Actively managed funds with style tilts

Name of Fund	Alpha	S(0)	Market tilt	Size tilt	Value tilt	R Square
ABLIF	-1.034	-0.0785	0.756	-0.254	-0.542	75%
			0.548	1.578	1.254	
ABLSF	-0.404	-0.0001	0.824	-0.004	-0.264	71%
			1.784	1.0874	0.956	
AKDAIF	0.5932	0.0485	0.72	0.54	0.24	46%
			0.004	0.0045	0.0078	
AKDITF	-1.25	-2.245	0.0045	-1.21	-1.15	35%
			1.254	0.547	0.541	
AKDOF	1.452	1.024	0.87	0.85	0.45	65%
			0.000	0.0045	0.001	
AMMF	0.842	0.265	0.78	0.54	0.51	48%
			0.005	0.0024	0.004	
MCF	0.487	0.245	0.67	0.46	0.64	39%
			0.001	0.0045	0.0087	
MIF	-0.054	-0.003	0.754	0.002	-0.24	48%
			0.587	0.874	0.544	
MIIF	-0.125	-0.084	0.38	-0.45	-0.45	25%
			1.245	2.587	2.145	
AGAF	-0.152	-0.024	0.45	-0.78	-0.12	23%
			0.897	0.158	0.24	
AGIMF	1.784	0.451	0.68	0.87	0.56	40%
			0.004	0.0045	0.0001	
AGIF	2.154	0.845	0.71	0.76	0.63	67%
			0.458	0.158	0.245	
AGVF	-0.125	-0.005	0.31	-0.452	-0.142	28%
			2.015	1.254	1.254	
MCBCMPF	-0.785	-0.004	0.25	-0.752	-0.456	36%
			1.245	1.785	0.547	
MCBDAF	2.456	0.845	0.57	0.68	0.71	73%
			0.456	0.24	0.985	
MCBDCF	2.652	0.985	0.46	0.54	0.47	67%
			0.0004	0.0094	0.006	
MCBDSF	1.458	0.005	0.67	0.75	0.69	71%
			0.0045	0.0004	0.0054	
MPSF	0.0458	0.0001	0.43	0.39	0.28	58%
			0.001	0.008	0.0045	
MPSF	0.0004	0.0001	0.65	0.43	0.51	72%
			0.004	0.0045	0.0056	
PCMF	-0.0045	-0.0006	0.52	-0.152	-0.65	54%
			1.25	2.271	1.045	
PCMF	3.154	1.258	0.78	0.84	0.76	82%
			0.002	0.0045	0.025	
PIEF	-0.34	-0.0056	0.54	-0.154	-0.258	46%
			0.548	0.245	1.254	
PIF	-0.021	-0.0001	0.46	-0.045	-0.21	52%
			0.548	0.874	0.004	
PIEIAAF	-0.124	-0.0002	0.39	-0.58	-0.18	40%
			0.004	0.0045	0.008	
PPF	-0.004	-0.0005	0.43	-0.15	-0.56	36%
			0.000	0.0005	2.021	
PSMF	0.454	0.004	0.61	0.47	0.52	57%

			0.000	0.0045	0.024	
PSA	0.125	0.0245	0.58	0.69	0.73	68%
			0.000	0.000	0.000	
AAAF	1.245	0.425	0.60	0.75	0.68	75%
			0.0045	0.0045	0.000	
AHYS	-0.245	-0.045	0.45	-0.45	-0.34	37%
			0.000	1.125	0.000	
AIF	-0.751	-0.004	0.31	-0.023	-0.14	46%
			0.0000	0.000	1.547	
AIIF	2.045	0.523	0.64	0.81	0.75	78%
			0.0000	0.0045	0.000	
AISF	1.025	0.254	0.52	0.47	0.39	64%
			0.0000	0.0004	0.0000	
ASMF	0.854	0.0004	0.46	0.51	0.64	0.59%
			0.0000	0.0001	0.000	
BCRSF	1.025	0.0008	0.46	0.58	0.46	53%
			0.000	0.0000	0.000	
DIF	-0.045	-0.155	0.66	-0.014	0.025	48%
			0.0002	0.0001	0.000	
DIF	-0.0045	-0.0254	0.23	-0.001	-0.0456	39%
			0.000	0.000	0.0000	
FAAF	-0.001	-0.257	0.54	-0.002	0.0021	45%
			0.0032	0.015	0.000	
FBGF	0.021	0.0004	0.45	0.34	0.68	61%
			0.0002	0.000	0.000	
FIGF	0.035	0.0025	0.47	0.46	0.54	63%
			0.000	0.000	0.000	
FSGF	0.45	0.045	0.64	0.48	0.53	38%
			0.0002	0.000	0.0000	
FHIF	0.458	0.21	0.42	0.56	0.61	76%
			0.0006	0.0005	0.0000	
HIF	1.245	0.58	0.64	0.81	0.480	69%
			0.000	0.0000	0.0002	
HMAF	2.45	0.97	0.74	0.68	0.48	47%
			0.005	0.0005	0.0005	
HSF	0.001	0.0003	0.46	0.39	0.49	51%
			0.0045	0.0004	0.0058	
IGIIF	3.145	1.325	0.58	0.84	0.72	75%
			0.0004	0.0004	0.0005	
IGISF	0.025	0.0035	0.46	0.38	0.56	52%
			0.004	0.0078	0.002	
JSAAA	0.1025	0.0012	0.37	0.48	0.59	47%
			0.001	0.002	0.005	
JSAIF	0.001	0.00045	0.29	0.45	0.35	53%
			0.007	0.0045	0.0078	
JSFOF	0.25	0.045	0.46	0.63	0.58	64%
			0.000045	0.0035	0.0045	
JSIF	1.023	0.86	0.56	0.74	0.82	61%
			0.0025	0.0024	0.0078	
JSIF	1.25	0.89	0.64	0.74	0.55	73%
			0.004	0.000	0.000	
JSKSEIF	2.25	0.91	0.56	0.45	0.55	48%
			0.0047	0.0007	0.0005	
JSLCF	-0.24	0.0005	0.26	-1.35	-0.98	16%
			0.05	0.054	0.006	

UTOF	-0.10	-0.025	0.45 0.005	-0.75 0.458	-0.23 0.004	34%
CDF	2.125	0.964	0.29 0.0045	0.74 0.0024	0.56 0.0078	70%
KASBAAF	-0.015	0.005	0.43 0.554	-0.24 0.547	-0.43 0.004	34%
KASBIOF	0.025	0.003	0.43 0.004	0.65 0.244	0.46 0.004	52%
KASBIIOF	1.26	0.65	0.62 0.0062	0.84 0.0048	0.46 0.0024	65%
KASBSMF	2.245	0.864	0.59 0.95	0.74 0.0078	0.56 0.045	70%
NAMCOIF	2.540	0.256	0.46 0.006	0.67 0.0007	0.58 0.0085	62%
NIUT	-0.65	-0.005	0.36 2.045	-0.16 0.005	-0.28 0.007	56%
NAFAIF	-0.023	-0.0004	0.64 0.0001	-0.45 0.0000	-0.39 2.548	43%
NAFAIOF	0.024	0.005	0.58 0.0078	0.51 0.0069	0.61 0.000	49%
NAFAIMAF	-0.145	-0.005	0.49 4.245	-0.15 0.000	-0.21 2.05	46%
NAFAMAF	1.165	0.5425	0.56 2.054	0.66 0.000	0.58 0.125	66%
NAFASF	-0.015	0.005	0.43 0.003	-0.24 0.000	-0.43 2.45	34%
POIAF	0.025	0.003	0.43 0.0078	0.65 0.005	0.46 0.045	52%
UCIF	1.26	0.65	0.62 0.0000	0.84 0.0000	0.46 0.000	65%
UGIF	-1.023	0.86	0.56 0.000	-0.74 0.254	-0.82 0.000	61%
USAF	-0.25	-0.001	0.56 0.000	-0.45 2.045	-0.025 0.004	48%

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